# 9

# Mechanical Properties of Fluids

*When a parachute jumper jumps from a flying plane, he descends very slowly in the air. This is an interesting real-life application of Stoke's law. The man falls with gravitational acceleration at first, but the acceleration quickly decreases until the parachute is fully opened. Because of the viscosity of air, the man's acceleration eventually becomes zero, and he falls at a constant terminal speed.*

## Topic Notes

- *Pressure and its Properties*
- *Viscosity and Surface Tension*

# PRESSURE AND ITS PROPERTIES 1

## TOPIC 1

## PRESSURE DUE TO A FLUID COLUMN

Fluids are defined as liquids and gases that can flow together. In other words, fluids are substances that can flow. Fluid is incompressible (*i.e.*, its density is independent of pressure variation and remains constant) and non-viscous (*i.e.*, the two liquid surfaces in contact are not exerting any tangential force on each other).

### Pressure

Pressure P is the normal force per unit area defined at each point.

$$P = \frac{F}{A}$$

Its SI unit of pressure is the Pascal

and 1 Pascal = 1 $\frac{N}{m^2}$ Fluid force acts perpendicular to any surface in the fluid, regardless of its orientation. As a result, pressure is a scalar quantity with no inherent direction. The force applied perpendicular to the surface of an object per unit area over which that force is distributed. The thrust experienced per unit area of the surface of a liquid at rest is called pressure. When a liquid is in equilibrium, the force acting on its surface is perpendicular everywhere. The pressure P is the same at the same horizontal level. The pressure at any point in the liquid depends on the depth ($h$) below the surface, density of liquid and acceleration due to gravity.

**Example 1.1: A 50 kg girl wearing high heel shoes balances on a single heel. The heel is circular with a diameter 1.0 cm. What is the pressure exerted by the heel on the horizontal floor? [NCERT]**

**Ans.** Given, Mass of the girl, $m = 50$ kg

Diameter of the heel, $d = 1$ cm $= 0.01$ m

Radius of the heel, $r = \frac{d}{2} = 0.005$ m

Area of the heel, $\pi r^2 = \pi (0.005)^2$

$= 7.85 \times 10^{-5}\ m^2$

Force exerted by the heel on the floor:

$$F = mg = 50 \times 9.8 = 490\ N$$

Pressure exerted by the heel on the floor:

$$P = \frac{Force}{Area} = \frac{490}{7.85} \times 10^{-5}$$

$$= 6.24 \times 10^6\ Nm^{-2}$$

Therefore, the pressure exerted by the heel on the horizontal floor is $6.24 \times 10^6\ Nm^{-2}$.

### Atmospheric Pressure

It is the atmospheric pressure of the Earth. The average atmospheric pressure at sea level is 1 atmosphere (atm), which is equal to $1.013 \times 10^5$ Pa.

The excess pressure above atmospheric pressure is referred to as gauge pressure, while total pressure is referred to as absolute pressure.

A barometer is a device that measures atmospheric pressure, whereas a U-tube manometer or simply a manometer, measures gauge pressure.

**Example 1.2: Explain why;**

**(A) The blood pressure in humans is greater at the feet than at the brain?**

**(B) Atmospheric pressure at a height of about 6 km decreases to nearly half of its value at sea level, though the height of the atmosphere is more than 100 km.**

**(C) Hydrostatic pressure is a scalar quantity even though the pressure is force divided by area. [NCERT]**

**Ans.** (A) The pressure of a liquid is given by the relation: $P = h\rho g$

It can be inferred that pressure is directly proportional to height. Hence, the blood pressure in human vessels depends on the height of the blood column in the body. The height of the blood column is more at the feet than it is at the brain. Hence, the blood pressure at the feet is more than it is at the brain.

(B) Density of air is maximum near the sea level. The density of air decreases with an increase in height from the surface. At a height of about 6 km, density decreases to nearly half of its value at sea level. Atmospheric pressure is proportional to density. Hence, at a height of 6 km from the surface, it decreases to nearly half of its value at sea level.

(C) When force is applied to a liquid, the pressure in the liquid is transmitted in all directions. Hence, hydrostatic pressure does not have a fixed direction and it is a scalar physical quantity.

**Example 1.3** Toricelli's barometer used mercury. Pascal duplicated it using French wine of density 984 kg m$^{-3}$. Determine the height of the wine column for normal atmospheric pressure. [NCERT]

**Ans.** Density of mercury, $\rho_1 = 13.6 \times 10^3$ kg/m$^3$

Height of the mercury column, $h_1 = 0.76$ m

Density of french wine, $\rho_2 = 984$ kg/m$_3$

Height of the french wine column = $h_2$

Acceleration due to gravity, $g = 9.8$ m/s$^2$

The pressure in both columns is equal, *i.e.,*

Pressure in the mercury column

= Pressure in the french wine column

$\rho_1 h_1 g = \rho_2 h_2 g = 10.5$ m

Hence, the height of the French wine column for normal atmospheric pressure is 10.5 m.

## Pascal's Law

According to Pascal's Law, the pressure applied to an enclosed liquid is transmitted undiminished to every portion of the liquid and the walls of the containing vessel.

Hydraulic system works on Pascal's law. If the force is exerted on any area then the ratio will be the same at all cross-sections.

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

A change in pressure applied to an enclosed fluid is transmitted unequally to all parts of the fluid as well as the walls of the containing vessel.

There are a lot of practical applications of Pascal's law. One such application is the hydraulic lift.

## Variation of Pressure with Depth

Consider a liquid-filled vessel. Because the liquid is in equilibrium, every volume element of the fluid is in equilibrium as well. Consider a single volume element in the form of a cylindrical column of liquid with height $h$ and cross-section area A.

The various forces acting on the cylindrical column of liquid are:

(1) Force, $F_1 = P_1A$ acting vertically downward on the top face of the column. $P_1$ is the pressure of the liquid on the top face of the column and is known as atmospheric pressure.

(2) Force, $F_2 = P_2A$ acting vertically upward at the bottom face of the cylindrical column. $P_2$ is the pressure of the liquid on the bottom face of the column.

(3) Weight, $W = mg$ of the cylindrical column of the liquid acting vertically downward. Since the cylindrical column of the liquid is in equilibrium, the net force acting on the column is zero.

*i.e.,* $F_1 + W - F_2 = 0$

$$P_1A + mg - P_2A = 0$$

$$P_1A + mg = P_2A$$

$$P_2 = P_1 + \frac{mg}{A} \quad \text{...(i)}$$

Mass of the cylindrical column of the liquid

$m$ = volume × density of the liquid

= area of cross-section × height × density

= $Ah\rho$

Substituting this value of mass in equation (i),

$$P_2 = P_1 + \frac{Ah\rho g}{A}$$

or, $$P_2 = P_1 + h\rho g \quad \text{.... (ii)}$$

$P_2$ is the absolute pressure at depth $h$ below the free surface of the liquid. Equation (ii), shows that the absolute pressure at depth $h$ is greater than the atmospheric pressure ($P_1$) by an amount equal to $h\rho g$.

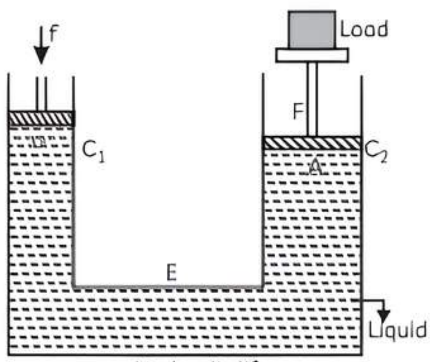
$$(P_2 - P_1) = h\rho g$$

which is the difference of pressure between two points separated by a depth $h$. It is called gauge pressure.

## Applications of Pascal's Law

### Hydraulic Lift

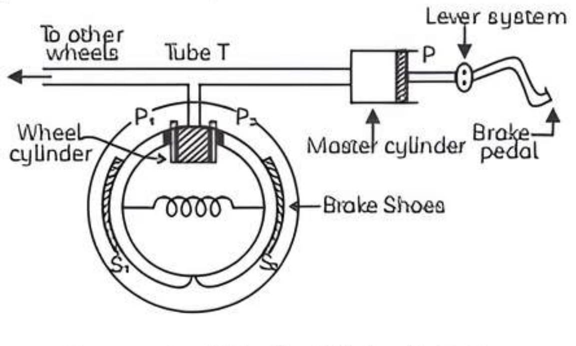
Hydraulic lift is used to lift heavy loads. It is a force multiplier. Here $C_1$ and $C_2$ are two cylinders of different areas of cross – section. They are connected to each other with a pipe E each cylinder is provided with airtight frictionless piston. Let $a$ and A be the areas of cross – section of the pistons in $C_1$ and $C_2$ respectively, where $a$ <<< A. The cylinders are filled with an incompressible liquid. The small force applied on the smaller piston of $C_1$ will be appearing as a very large force on the larger a piston of $C_2$. As a result of it, a heavy load placed on the larger piston is easily lifted upwards.



Hydraulic lift

## Hydraulic Brake

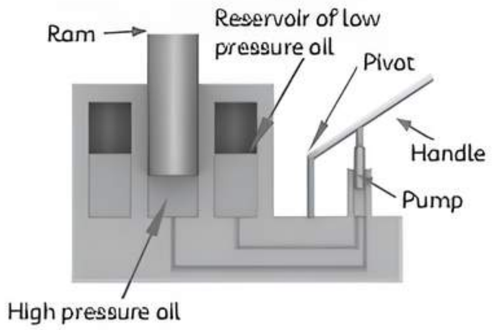
The piston is connected to the brake pedal through the lever system. The area of the cross-section of the wheel cylinder is greater than that of master cylinder. When a little force is applied on the pedal with foot, the master piston (P) moves inside. The pressure thus caused is transmitted undiminished to the other pistons, ($P_1$ and $P_2$) which are directly connected to the wheels. These pistons are of larger cross-section as compared to the master piston. Applying Pascal's law, a large force acts on these pistons that move out and expand the brake shoes. Thus, the braking force produced at wheels is quite large as compared to the little force applied at the brake pedal.




*Constructional details of Hydraulic Brakes*

**Example 1.4: Case Based:**

*The force generated in a hydraulic system depends on the size of the pistons. If the smaller of the two pistons is two inches and the larger piston is six inches, or three times as large, the amount of force created will be nine times greater than the amount of force from the smaller piston. One hundred pounds of force by a small piston will be able to lift 900 pounds.*




**(A)** **A hydraulic lift is designed to lift cars with a maximum mass of $4 \times 10^3$ kg. The area of cross-section of the piston carrying the load is $5 \times 10^{-2}$ $m^2$. How much pressure the smaller piston will bear?**

**(B)** **To lift an automobile of 2000 kg, a hydraulic pump with a large piston 900 $cm^2$ in the area is employed. Calculate the force that must be applied to pump a small piston of area 10 $cm^2$ to accomplish this.**

**(C)** **Assertion (A): To empty an oil tank, two holes are made.**

**Reason (R): Oil will come out of two holes, so it will be emptied faster.**

**(a) Both A and R are true and R is the correct explanation of A.**

**(b) Both A and R are true and R is not correct explanation of A.**

**(c) A is true but R is false.**

**(d) A is false and R is also false.**

**[Delhi Gov. QB 2022]**

**(D)** **The statement, "The pressure in a fluid at rest is the smallest at all points if they are at the same height" represents which of the following?**

**(a) Bernoulli's principle**

**(b) Archimedes principle**

**(c) Pascal's law**

**(d) Boyle's law**

**(E)** **Pressure on a swimmer, 10 m below the lake surface water, will be:**

| | |
|---|---|
| **(a) 2 atm** | **(b) 5 atm** |
| **(c) 7 atm** | **(d) 0.2 atm** |

**Ans. (A)** Maximum pressure = $\frac{\text{Maximum force}}{\text{Area}}$

Here, Maximum mass,

$$m = 4 \times 10^3 \text{ kg}$$

Maximum force,

$$mg = 4 \times 10^3 \times 9.8$$
$$= 39.2 \times 10^3 \text{ N}$$

And Area, $A = 5 \times 10^{-2} \text{ m}^2$

$$\text{Maximum pressure} = \frac{\text{Maximum force}}{\text{Area}}$$

$$\text{Maximum pressure} = \frac{39.2 \times 10^3}{5 \times 10^{\ 2}}$$
$$= 7.84 \times 10^5 \text{ Nm}^{-2}$$

**(B)** As we know $P = v \frac{v_1 + v_2}{4}$

Here, $F = mg = 2000 \times 9.8$

$= 19600$ N

$A = 900 \text{ cm}^2$

$= 900 \times 10^{-4} \text{m}^2$

$$P = \frac{F}{A} = \frac{19600 \text{ N}}{900 \times 10^{-4} \text{m}^2}$$
$$= 2.2 \times 10^5 \text{ Nm}^{-2}$$

Applied Force $F = P \times A$

$= 2.2 \times 10^5 \times 10 \times 10^{-4}$

$= 220$ N

**(C)** *(c) A is true but R is false.*

**Explanation:** When two holes are made in the tin, air keeps on entering through the other hole. Due to this pressure inside the tin does not become less than atmospheric pressure which happens if only one hole is made.

**(D)** *(c) Pascal's law*

**Explanation:** Bernoulli's principle states that the total mechanical energy of the moving fluid comprising the gravitational potential energy of elevation, the energy associated with the fluid pressure and the kinetic energy of the fluid motion, remains constant.

Archimedes principle states that a body immersed in a fluid is subjected to an upwards force equal to the weight of the displaced fluid. This is the first condition of equilibrium. We consider that the upward force, called the force of buoyancy, is located in the centre of the submerged hull that we call the centre of buoyancy.

Pascal's principle, also called Pascal's law, in fluid (gas or liquid) mechanics, is a statement that in a fluid at rest in a closed container, a pressure change in one part is transmitted without loss to every portion of the fluid and the walls of the container. The principle was first enunciated by the French scientist Blaise Pascal.

Boyle's law, also called Mariotte's law, is a relation concerning the compression and expansion of a gas at constant temperature. This empirical relation, formulated by the physicist Robert Boyle in 1662, states that the pressure (P) of a given quantity of gas varies inversely with its volume (V) at constant temperature; *i.e.*, in equation form, $\rho V = k$, (constant)

**(E)** *(a)* 2 *atm*

**Explanation:** Let Atmospheric pressure

$$P = 1.05 \times 10^5\ Pa$$
$$h = 10\ m$$
$$\rho = 1000\ kg/m^3$$
$$g = 10\ m/s^2$$
$$P = P_0 + \rho gh$$
$$= 1.05 \times 10^5\ Pa + 1000\ m/s^2 \times 10\ m/s^2 \times 10\ m$$
$$= 2.01 \times 10^5\ Pa = 2\ atm$$

## TOPIC 2

# BUOYANCY

The upward force acting on the body is immersed in a fluid called upward thrust or buoyant force and the phenomenon is called buoyancy. The buoyant force acts at the center of buoyancy which is the centre of gravity of the liquid displaced by the body during immersion.

### Archimedes Principle

When a body is immersed in a fluid, the fluid exerts a contact force on the body. The sum of all these contact forces is known as buoyant force (upthrust).

This force is called buoyant force and acts vertically upwards (opposite to the weight of the body) through the centre of gravity of the displaced fluid.

$$F = V\sigma g$$

where, $F$ = weight of fluid displaced by the body.

$V$ = volume of liquid displaced

$\sigma$ = density of liquid.

Apparent decrease in weight of body = upthrust

= weight of liquid displaced by the body.

### Floatation

A body floats in a liquid if its average density is less than the liquid's density. The weight of the liquid displaced by the immersed body part must be equal to the body's weight. The body's centre of gravity and centre of buoyancy must be on the same vertical line.

The law of floatation states that a body will float in a liquid if the weight of the liquid displaced by the immersed part of the body is at least equal to or greater than the weight of the body.

Using Archimedes principle,

the relative density of the body can be determined as:

$$\text{Relative density} = \frac{\text{Density of body}}{\text{Density of pure water at } 4°C}$$

$$\text{Relative density} = \frac{\text{Weight of body}}{\text{Weight of equal volume of water}}$$

**Example 1.5: A vertical offshore structure is built to withstand maximum stress of $10^9$ Pa. Is the structure suitable for putting up on top of an oil well in the ocean? Take the depth of the ocean to be roughly 3 km, and ignore ocean currents.**

**[NCERT]**

**Ans.** The maximum allowable stress for the structure,

$$P = 10^9\ Pa$$

Depth of the ocean,

$$d = 3\ km = 3 \times 10^3\ m$$

Density of water, $\rho = 10^3\ kg/m^3$

Acceleration due to gravity,

$$g = 9.8\ m/s^2$$

The pressure exerted because of the seawater at depth, $P = \rho dg$

$$= 3 \times 10^3 \times 10^3 \times 9.8$$
$$= 2.94 \times 10^7\ Pa$$

The maximum allowable stress for the structure ($10^9$ Pa) is greater than the pressure of the seawater ($2.94 \times 10^7$ Pa). The pressure exerted by the ocean is less than the pressure that the structure can withstand. Hence, the structure is suitable for putting up on top of an oil well in the ocean.

**Example 1.6: The atmospheric pressure drops to nearly half its value at sea level at a height of 6 km, despite the fact that the atmosphere is more than 100 km above the ground. Explain why?** **[NCERT]**

**Ans.** This is because the atmospheric pressure does not decrease linearly with height. The atmospheric pressure decreases exponentially with height as

$$P = P_0 e^{-y/y_0}$$

## TOPIC 3

# FLUID DYNAMICS

## Steady Flow (Streamline Flow)

Streamline flow occurs when a liquid (fluid) flows in such a way that each particle of the liquid passing a point moves along the same path and has the same velocity as its predecessor. It is also known as laminar flow.

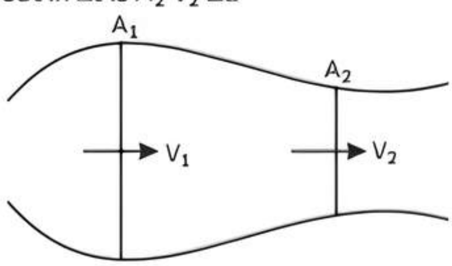
A particle's path is referred to as a streamline. The trajectories of fluid particles are represented by streamlines. It is a curve (or a straight line) drawn in such a way that the tangent to it at a point gives the flow direction at that point. Two streamlines will never intersect. If they cross, the fluid particle flowing through the point of intersection will be able to flow in two different directions at the same time, which is impossible. The greater the liquid velocity, the closer the streamline, and vice versa.

### Line of Flow

It is the path taken by a particle in a flowing liquid. In case of a steady flow, it is called streamline. Two streamlines can never intersect.

### Equation of Continuity

In a time $\Delta t$, the volume of liquid entering the tube of flow in a steady flow is $A_1V_1 \Delta t$. The same volume must flow out, as the liquid is incompressible. The volume flowing out in $\Delta t$ is $A_2 V_2 \Delta t$.



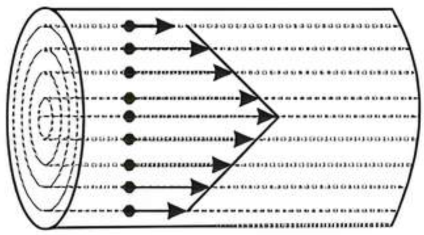

$$A_1V_1 = A_2 V_2$$

$$\text{mass flows rate} = \rho AV$$

Where $\rho$ is the density of the liquid.

## Laminar Flow

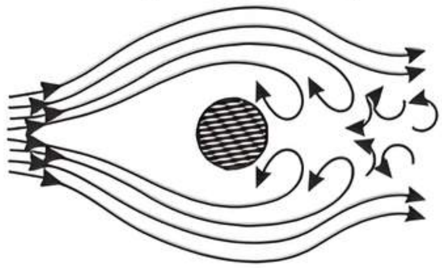
If the liquid flows over a horizontal surface in the form of layers of different velocities, then the flow of liquid is known as laminar flow. The particles of one layer in laminar flow do not enter into another layer.



Laminar flow of liquid

## Turbulent Flow

When a liquid moves with a velocity greater than its critical velocity, the motion of the particles of liquid is disorderly or irregular. Such flow is known as turbulent flow. The speed of the flow is quite high and boundary surfaces cause changes in the velocity of the flow.



Turbulent flow of liquid

**Important**

➥ *The critical velocity is the velocity of liquid flow, up to which its flow is streamlined and above which its flow becomes turbulent.*

*It is given by* $v_c = \dfrac{k\eta}{\rho r}$

**Example 1.7:** **Two pipes of diameters $d_1$ and $d_2$ converge to form a pipe of diameter $2d$. If the liquid flows with a velocity of $v_1$ and $v_2$ in the two pipes, what will be the flow velocity in the third pipe?**

(a) $v_1 + v_2$

(b) $\dfrac{v_1 + v_2}{2}$

(c) $\dfrac{v_1 + v_2}{4}$

(d) $2(v_1 + v_2)$ **[NCERT]**

**Ans.** *(c)* $\dfrac{v_1 + v_2}{4}$

**Explanation:** According to the continuity equation,

$$\sum_{i=1}^{n} A_i v_i = \sum_{o=1}^{n} A_o v_o$$

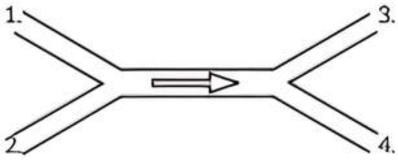
Where A, represents flow area, $v$ represents flow velocity, $i$ is for inlet conditions and O is for outlet conditions. Thus,

$$A_1 v_1 + A_2 v_2 = Av$$

$$d_1^2 v_1 + d_2^2 v_2 = d^2 v$$

$$v = \frac{v_1 + v_2}{4}$$

**Example 1.8:** In a water supply system, water flows in from pipes 1 and 2 and goes out from pipes 3 and 4 as shown. If all the pipes have the same diameter, which of the following must be correct?




(a) The sum of the flow velocities in 1 and 2 is equal to that in 3 and 4

(b) The sum of the flow velocities in 1 and 3 is equal to that in 2 and 4

(c) The sum of the flow velocities in 1 and 4 is equal to that in 2 and 3

(d) The flow velocities in 1 and 2 is equal to that in 3 and 4

**Ans.** *(a) The sum of the flow velocities in 1 and 2 is equal to that in 3 and 4*

**Explanation:** According to the continuity equation,

$$\sum_{i=1}^{n} A_i v_i = \sum_{o=1}^{n} A_o v_o$$

where A represents flow area, $v$ represents flow velocity, $i$ is for inlet conditions and $O$ is for outlet conditions.

$$A_1 v_1 + A_2 v_2 = A_3 v_3 + A_4 v_4$$

Since, $d_1 = d_2 = d_3 = d_4$

$$v_1 + v_2 = v_3 + v_4$$

# OBJECTIVE Type Questions

[ 1 mark ]

## Multiple Choice Questions

**1.** A student can reduce the pressure in his lungs to 750 mm Hg (density=13.6 g/cm$^3$) by sucking through a straw. He can drink water from a glass using the straw up to a maximum depth of:

(a) 10 cm (b) 75 cm

(c) 13.6 cm (d) 1.36 cm

**Ans.** *(c) 13.6 cm*

**Explanation:** Pressure difference between atmosphere and lungs $= h\rho_w g$

$$76 \times \rho_w \times g - 75 \times \rho_{Hg} \times g = h\rho_w g$$

$$1 \times 13.6 = h \times 1$$

$$h = 13.6 \text{ cm}$$

### Related Theory

→ *Pressure due to liquid on a vertical wall is different at different depths, so average fluid pressure on a sidewall of a container*

$$= \text{mean pressure} = \frac{h\rho g}{2}$$

*Here $h$ = height of wall*

**2.** The approximate depth of an ocean is 2700 m. The compressibility of water is 45.4 × 10$^{-11}$ Pa$^{-1}$ and the density of water is 10$^3$ kg/m$^3$. What fractional compression of water will be obtained at the bottom of the ocean?

(a) 1.0 × 10$^{-2}$ (b) 1.2 × 10$^{-2}$

(c) 1.4 × 10$^{-2}$ (d) 0.8 × 10$^{-2}$

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**Ans.** *(b) 1.2 × 10$^{-2}$*

**Explanation:** Compressibility is given as

$$k = \frac{\frac{\Delta V}{V}}{\Delta P}$$

Substituting values

$$\Delta P = \rho g h \text{ Pa.}$$

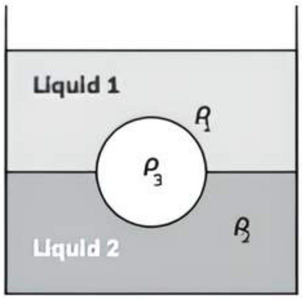
$$k = 45 \times 10^{-11} \text{ Pa}^{-1}$$

$$\frac{\Delta V}{V} = 45 \times 10^{-11} \times \Delta P$$

$$\frac{\Delta V}{V} = 45 \times 10^{-11} \times 10^3 \times 10 \times 2700$$

$$\frac{\Delta V}{V} = 1.2 \times 10^{-2}$$

**3.** A bucket contains two non-mixing liquids 1 and 2 with densities $\rho_1$ and $\rho_2$ respectively. A solid ball made of $\rho_3$, density material is dropped into the jar. It reaches equilibrium in the position depicted in the figure.




Which of the following statements is true for $\rho_1$, $\rho_2$ and $\rho_3$?

(a) $\rho_3 < \rho_1 < \rho_2$ (b) $\rho_1 > \rho_3 > \rho_2$
(c) $\rho_1 < \rho_2 < \rho_3$ (d) $\rho_1 < \rho_3 < \rho_2$

**Ans.** *(d) $\rho_1 < \rho_3 < \rho_2$*

**Explanation:** Denser objects tend to settle down and lighter objects tend to float up.

Clearly, $\rho_1 < \rho_2$.

Since the ball does not rise to the surface,

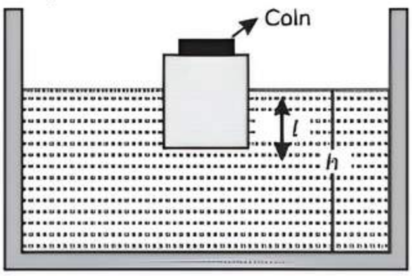
$$\rho_1 > \rho_1$$

Neither does the ball settle at the bottom,

$$\rho_3 < \rho_2$$

Hence, $\rho_1 < \rho_3 < \rho_2$

**4.** As shown in the figure, a timber block with a coin on top floats in water. There are distances *l* and *h* shown. After a while, the coin falls into the water, then:




(a) *l* decreases and *h* increases
(b) *l* increases and *h* decreases
(c) both *l* and *h* increases
(d) both *l* and *h* decreases

**Ans.** *(d) both l and h decreases*

**Explanation:** Density of the coin is more so it was applying considerable force on wood before falling into the water. But once it'll fall, wood can't apply the same force alone, so the wood will come a little up as a result *l* will decrease.

When a coin falls in the water, the wood goes up, so displaced volume decreases and as the coin has higher density so its volume is less and hence it doesn't displace liquid that much. As a result, *h* will also decrease.

**5.** Along a streamline:

(a) the velocity of a fluid particle remains constant.
(b) the velocity of all fluid particles crossing a given position is constant.
(c) the velocity of all fluid particles at a given instant is constant.
(d) the speed of a fluid particle remains constant. [NCERT Exemplar]

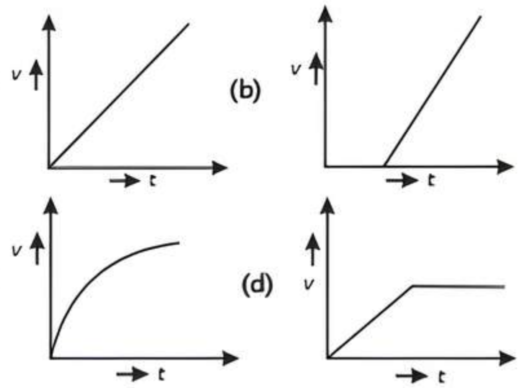
**Ans.** *(b) the velocity of all fluid particles crossing a given position is constant.*

**Explanation:** In streamlined flow, the speed at a point in a cross-section is always constant because Av = constant.

### Related Theory

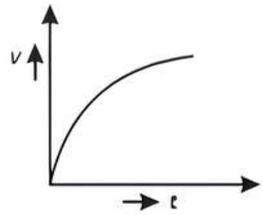
*When water falls from a tap, the velocity of falling water under the action of gravity will increase with distance from the tap (i.e. $v_2 > v_1$).*
*So in accordance with an equation of continuity, the cross-section of the water stream will decrease (i.e. $A_2 > A_1$) i.e. falling stream of water becomes narrower.*

**6.** A tall cylinder is filled with viscous oil. A round pebble is dropped from the top with zero velocity. From the plot shown in the figure, indicate the one that represents the velocity *v* of the pebble as a function of time (*t*).




[NCERT Exemplar]

**Ans.** *(c)*




**Explanation:** When the pebble is dropped from the top of a cylinder filled with viscous oil, the pebble falls under gravity with constant acceleration, but as it is dropped it enters in the oil and dragging force F = 6 $\pi\eta rv$ due to the viscosity of the oil so acceleration decreases from *g* to zero *i.e.*, velocity increases, but acceleration decreases, when acceleration decreased to zero, velocity becomes constant (terminal velocity).

**7.** Current flows at a velocity of 4 m/sec through a cylinder with a diameter of 8 cm, which is connected to a pipe with a diameter of 2 cm at its end tip. Determine the velocity of the water at its free end:

(a) 4 m/s (b) 8 m/s
(c) 32 m/s (d) 64 m/s

**Ans.** *(d) 64m/s*

**Explanation:** $A_1v_1 = A_2v_2$

Here, $(\pi(0.04)^2)(4) = (\pi(0.01)^2)v_2$

$v_2 = 64$ m/s

**8.** **An ideal fluid flow through a pipe of the circular cross-section made of two sections with diameters 2.5 cm and 3.75 cm. The ratio of the velocities in the two pipes is:**

(a) 9 : 4 (b) 3 : 2

(c) $\sqrt{3}:\sqrt{2}$ (d) $\sqrt{2}:\sqrt{3}$

[Delhi Gov. QB 2022]

**Ans.** *(a) 9 : 4*

**Explanation:** According to the equation of continuity $A_1v_1 = A_2v_2$

$$\frac{v_1}{v_2} = \frac{A_1}{A_2} = \frac{\frac{\pi D_2^2}{4}}{\frac{\pi D_1^2}{4}} = \left(\frac{D_2}{D_1}\right)^2$$

Here, $D_1 = 2.5$ cm, $D_2 = 3.75$ cm

$$\therefore \frac{v_1}{v_2} = \left(\frac{3.75}{2.5}\right)^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

## Assertion-Reason Questions

**Two statements are given one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below:**

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true and R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false and R is also false.

**9.** **Assertion (A):** The velocity increases, when water flowing in a broader pipe enters a narrow pipe.

**Reason (R):** According to the equation of continuity, product of the area of the cross-section and velocity is constant.

[Delhi Gov. QB 2022]

**Ans.** *(a) Both A and R are true and R is the correct explanation of A.*

**Explanation:** According to the equation of continuity, Av = constant, in a liquid streamline flow.

Where A is the cross-sectional area and *v* is the liquid flow velocity. When water flowing in a larger pipe enters a smaller pipe, the area of the cross-section of the water reduces and the velocity of the stream rises.

**10.** **Assertion (A):** A parachute descends slowly, whereas a stone dropped from the same height descends quickly.

**Reason (R):** The viscous force of air on a parachute is greater than the viscous force of a falling stone.

**Ans.** *(a) Both A and R are true and R is the correct explanation of A.*

**Explanation:** Gravity is pulling the man down while friction with the air slows him down. With the parachute out it adds more friction slowing him down because air resistance works against the very large surface area of the parachute. A free-falling skydiver that has not yet opened his parachute can make his downward speed vary between about 110 mph and 225 mph using air resistance. A spread-eagled position presents the maximum area and we fall the slowest.

**11.** **Assertion (A):** With increasing temperature, the angle of contact of a liquid decreases.

**Reason (R):** The surface tension of a liquid increases as its temperature rises.

**Ans.** *(a) Both A and R are true and R is the correct explanation of A.*

**Explanation:** Angle of contact decreases with the addition of impurities whereas it increases if the temperature is increased.

Surface tension causes an increase in cohesive forces. When the temperature rises, surface tension decreases and angle of contact increases.

**12.** **Assertion (A):** Surface tension of liquid decreases with an increase in temperature.

**Reason (R):** Cohesive force between liquid molecules increases with an increase in temperature.

**Ans.** *(c) A is true but R is false.*

**Explanation:** Surface tension decreases when temperature increases because cohesive forces decrease with an increase of molecular thermal activity. The influence of the surrounding environment is due to the adhesive action liquid molecules have at the interface. The impact of increasing the temperature of a liquid is to reduce the cohesive forces while simultaneously increasing the rate of molecular interchange. The former effect causes a decrease in the shear stress while the latter causes it to increase.

**13.** **Assertion (A):** The blood pressure in human is greater at the feet than at brain.

**Reason (R):** Pressure of liquid at any point is proportional to height, density of liquid and acceleration due to gravity.

[Delhi Gov. QB 2022]

**Ans.** (a) *Both A and R are true and R is the correct explanation of A.*

**Explanation:** The blood column in the human body is higher at the feet than at the brain.

As, $P = h\rho g$.

As a result, blood exerts greater pressure on the feet than on the brain.

**14.** **Assertion (A): The front of an automobile is designed to resemble the flow line pattern of the fluid through which it moves.**

**Reason (R): The shape of an automobile is streamlined to reduce the resistance provided by the fluid.**

**Ans.** (a) *Both A and R are true and R is the correct explanation of A.*

**Explanation:** The cars or planes are given such shape (known as streamlined shape) so that air friction is minimum. Rather the movement of air layers on the upper and lower side of the streamlined shape provides a lift which helps in increasing the speed of the car. When cars and planes move through air, their motion is opposed by air friction, which in turn, depends upon the shape of the body. It is due to this reason that cars or planes are given such shapes (known as streamlined shapes) so that air friction is minimum.

## CASE BASED Questions (CBQS)

[ 4 & 5 marks ]

**Read the following passages and answer the questions that follow:**

**15.** *Streamlines are the pathways or trajectories of fluid particles in a constant flow. The path of fluid particles is determined by the idea of fluid mechanics in physics. The liquid in a continuous stream is flowing, but the streamlines are stationary. The liquid speed is often high where streamlines swarm together; the liquid is somewhat calm where they open out. A streamline is a route that is perpendicular to the instantaneous velocity direction. Streamlines primarily operate in laminar flow rather than turbulent flow because laminar flow produces steady motion.*

**(A) What is streamlined and what are its properties?**

**(B) Do the two streamlines intersect each other in a streamline flow?**

**(C) How does water pressure one meter below the surface of a small pond compared to water pressure one meter below the surface of a huge lake?**

**Ans.** (A) The path taken by a fluid particle under a steady flow is streamline.

The properties of streamlined flow are:

(1) The tangent at any point in the streamline gives the direction of the fluid velocity at that point.

(2) Two streamlines never intersect each other.

(B) A tangent drawn from any point on the streamline indicates the direction of flow of the liquid. As two tangents (different) at a point will indicate two directions of fluid flow which is impossible. Hence, the two streamlines do not cross each other in streamline flow.

(C) Liquid pressure is given by the formula,

Liquid pressure = weight density of the liquid × height

Liquid pressure is independent of the area of the liquid at a particular depth. So, water pressure 1 m below the surface of a small pond will be the same as that due to water pressure 1 m below the surface of a huge lake.

**16.** *A hydraulic lift is a device for moving objects using force created by pressure on a liquid inside a cylinder that moves a piston upward. Incompressible oil is pumped into the cylinder, which forces the piston upward. When a valve opens to release the oil, the piston lowers by gravitational force. The principle for hydraulic lifts is based on Pascal's law for generating force or motion, which states that pressure change on an incompressible liquid in a confined space is passed equally throughout the liquid in all directions. The concept of Pascal's law and its application to hydraulics can be seen in the example below, where a small amount of force is applied to an incompressible liquid on the left to create a large amount of force on the right.*

**(A) The rectangular vessel's base measures 10 cm × 18 cm. Mixture is pumped into the**

hole to a depth of 4 cm. What is the base's pressure? What is the base's thrust? [$g$ = 10 m/s$^2$.]

(a) 7.2 N (b) 8.3 N
(c) 5.7 N (d) 7.5 N

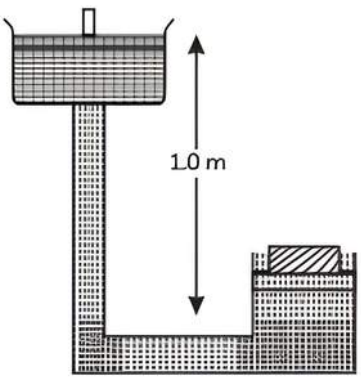
(B) A girl weighs 50 kg and balances on a single heel while wearing high heel shoes. The heel is circular, with a 1 cm diameter. How much pressure does the heel put on the horizontal floor?

(a) $8.24 \times 10^6$ Nm$^{-2}$
(b) $6.24 \times 10^6$ Nm$^{-2}$
(c) $10.24 \times 10^6$ Nm$^{-2}$
(d) $2.24 \times 10^6$ Nm$^{-2}$

(C) A hydraulic automobile lift is designed to lift vehicles weighing up to 3000 kg. The piston carrying the load has a cross-sectional area of 425 cm$^2$. What is the maximum pressure that the smaller piston must withstand?

(a) $6.92 \times 10^5$ Nm$^{-2}$
(b) $6.02 \times 10^4$ Nm$^{-2}$
(c) $6.92 \times 10^3$ Nm$^{-2}$
(d) $6.92 \times 10^2$ Nm$^{-2}$

(D) A hydraulic press with a larger piston of diameter 30 cm at a height of 1.0 m above the smaller piston of diameter 5 cm is shown in the figure. The smaller piston has a mass of 10 kg. What is the force that the larger piston exerts on the load? Oil in the press has a density of 750 kg/m$^3$.




(a) 3000.0 N (b) 3009.6 N
(c) 3009.3 N (d) 3009.2 N

(E) To lift an automobile of 5000 kg, a hydraulic lift with a large piston 800 cm$^2$ in area is employed. Calculate the force that must be applied to a small piston of area 5 cm$^2$ to achieve it.

(a) 612. 5 N (b) 306. 25 N
(c) 153.12 N (d) 408.56 N

**Ans.** (A) *(a) 7.2 N*

**Explanation:** Area of base

$= 10 \text{ cm} \times 18 \text{ cm}$

$= 180 \text{ cm}^2$

$= 180 \times 10^{-4} \text{ m}^2$

Depth of water, $h = 4 \text{ cm} = 4 \times 10^{-2}$ m

Density of water, $\rho = 10^3$ kg/m$^3$

Pressure on the base, $P = \rho h g$

$= 10^3 \times 4 \times 10^{-2} \times 10 = 400$ Pa

Thrust on the base, $F = P \times A$

$= 400 \times 180 \times 10^{-4} = 7.2$ N

(B) *(b) $6.24 \times 10^6$ Nm$^{-2}$*

**Explanation:** Force $F = 50 \times 9.8$ N

$$\text{Area, } A = \frac{\pi D^2}{4} = \frac{\pi(1\times10^{-2})^2}{4}$$

$$= \frac{3.14\times10^{\,4}\text{m}^2}{4}$$

$$\text{Pressure, } P = \frac{F}{A} = \frac{50\times9.8\times4}{3.14\times10^{-4}}$$

$= 6.24 \times 10^6$Nm$^{-2}$

(C) *(a) $6.92 \times 10^5$Nm$^{-2}$*

**Explanation:** Area of the cross-section of a bigger piston,

$A = 425 \text{ cm}^2 = 425 \times 10^{-4} \text{ m}^2$

Maximum load on the bigger piston,

$F = 3000 \times 9.8$ N

∴ Pressure on the bigger piston,

$$P = \frac{F}{A} = \frac{3000\times9.8}{425\times10^{-4}}$$

According to Pascal's law, the smaller piston would bear the same pressure as the bigger piston equal to $6.92 \times 10^5$ N/m$^2$

(D) *(b) 3009.6 N*

**Explanation:** Force on the smaller piston

$F = mg = 10 \times 9.8 = 98$ N

$$\text{Area of smaller piston} = \pi\left(\frac{D_2}{2}\right)^2 = \pi\frac{D_2^2}{4}$$

Pressure on the smaller piston

$$P = \frac{mg}{\pi D_1^2} = \frac{4\times9.8}{3.14\times0.0025}$$

Let F = force exerted on load by larger piston

Pressure by larger piston,

$$P = \frac{4\times F}{\pi D_1^2} = \frac{4F}{3.14\times 0.09} = 14.15\ F.Pa$$

Now pressure difference =

$$P = 750 \times 1 \times 9.8 = 7350\ Pa$$

Now 7350 = 49936.3 F – 14.15 F or

$$F = 3009.6\ N$$

### ⚠ Caution

➥ *Students should know that force is a vector, pressure is a scalar quantity because fluid pressure has no intrinsic direction of its own. Force due to fluid pressure always acts perpendicular to any surface in the fluid, no matter how that surface is oriented.*

(E) *(b)* 306.25 N

**Explanation:** Force on large piston

$$F_1 = mg$$
$$F_1 = 5000 \times 9.8\ N$$
$$= 4.9 \times 10^4\ N$$

Given area of large piston,

$$A_1 = 800\ cm^2$$

Pressure on the large piston

$$(P) = \frac{F_1}{A_1}$$
$$= \frac{4.9\times 10^4}{800\times 10^{-4}}$$
$$= 6.125 \times 10^5\ Pa$$

Since the liquid transmits pressure equally, therefore the pressure on the small piston will be equal to the pressure on the large piston.

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} = P$$

Given area of small piston

$$A_2 = 5\ cm^2$$

∴ Force applied on small piston

$$F_2 = P \times A_2$$
$$= 6.125\times 10^5 \times (5\times 10^{-4})$$
$$= 306.25\ N$$

## VERY SHORT ANSWER Type Questions (VSA)

[ 1 mark ]

**17. When a liquid weighs nothing, what shape does it take?**

**Ans.** When liquid weighs nothing, it is only the force of surface tension which acts on the liquid. Due to surface tension, the liquid has the tendency to acquire minimum surface. Since the surface area of a sphere is minimum, the liquid takes the shape of a sphere.

**18. Why is it difficult to stop bleeding from a cut in a human body at high altitude? [Delhi Gov. QB 2022]**

**Ans.** At high elevations, air pressure is low. Because of the higher pressure differential between blood pressure and atmospheric pressure, blood continues to flow out of the body, making it harder to halt bleeding from a wound in the body.

**19. Calculate the pressure on the body which is 30m below the surface of the water? [Diksha]**

**Ans.** As,

$$h = 30\ m$$
$$d = 1000\ kg/m^2$$
$$g = 9.8\ m/s^2$$
$$\rho_A = 1.0 \times 10^5 pa$$
$$P = \rho_A + hdg$$
$$P = 1.0 \times 10^5 + 30 \times 1000 \times 9.8$$
$$P = 4\ atm$$

**20. Iceberg floats in water with part of it submerged. What is the fraction of the volume of the iceberg submerged if the density of ice is, $\rho_I$ = 0.917 gm/cm³? [NCERT Exemplar]**

**Ans.** As iceberg is floating on the surface of the sea

∴ Weight of iceberg

= Weight of displaced liquid

$$V\rho_{ice}g = V'\rho_w g$$

V = Volume of iceberg

V' = Volume of the iceberg inside water or Volume of displaced water by iceberg

$$= \frac{\text{Volume of iceberg submerged}}{\text{Volume of iceberg}}$$

$$= \frac{V'}{V} = \frac{\rho_{ice}}{\rho_w} = \frac{0.917}{1}$$

Hence, 0.917 part of the iceberg body is submerged into water.

**21. Why are passengers advised to remove the ink from their pens before boarding an airplane?**

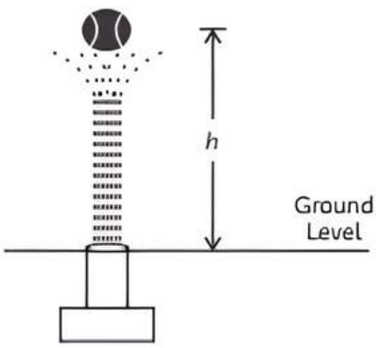
**Ans.** As the atmospheric pressure decreases with the height as ink inside the pen is filled at the atmospheric pressure existing on the surface of the earth, it tends to come out to equalize the pressure. This can spoil the clothes of the passengers, so they are advised to remove the ink from the pen.

## SHORT ANSWER Type-I Questions (SA-I)

[ 2 marks ]

**22. If you put a small tennis ball in a vertical jet of air or water, it will emerge to a certain height above the nozzle and stay there. Explain.**

**Ans.** Due to this high velocity of the jet of water, the pressure between the ball and the jet decreases. The greater (atmospheric) pressure on the other side of the ball pushes it against the jet and the ball remains suspended. The high velocity of water takes the ball upwards along with it and makes it spin. The ball does not fall as it is constantly pressed against the water jet due to the difference of pressure on the two sides of the ball.




Most of the trees are more than 0.6 m in height. So, capillary action alone cannot account for the rise of water in all other trees.

**23. Why do the ends of a glass tube become rounded on heating? [Diksha]**

**Ans.** When the glass tube is heated, it melts into a liquid. The surface of this liquid tends to have a minimum area. Now for a given volume, the area of the surface of a sphere is minimum. This is where the ends of a glass tube become a round on heating.

**24. How high would water rise in a building's pipes if the water pressure gauge reads 400 k Pa on the ground floor?**

**Ans.** Given P = 400 kPa = 400 × $10^3$ Pa

Using pressure, P = $h\rho g$

$$h = \frac{P}{\rho g}$$

$$= \frac{400 \times 10^3}{10^3 \times 9.8}$$

$$= 40.81$$

## SHORT ANSWER Type-II Questions (SA-II)

[ 3 marks ]

**25. The free surface of the oil in a tanker, at rest, is horizontal. If the tanker starts accelerating, the free surface will be tilted by an angle θ. If the acceleration is $a$ ms$^{-2}$, what will be the slope of the free surface? [NCERT Exemplar]**

**Ans.** Consider the tanker is pulled by force F which produces acceleration in the truck in the forward direction. Consider only a small element of mass δm. When a tanker is pulled by forward acceleration '$a$' then this element of mass also experiences it in a forward direction but due to the inertia of rest it tries to remain in rest due to the same acceleration in the backward direction as it is free not rigidly connected to the tanker.

Hence, forces acting on $\delta m$ are $F_1 = \delta m a$ in a horizontally backward direction due to the tanker's acceleration $g$.

$F_2 = \delta m g$ vertically downward due to gravity.

The resolution of components of $F_1$ and $F_2$ along a perpendicular to the inclined surface of oil are resolved $N$-normal reaction is balanced by component $\delta m a \sin\theta$ of $F_1$ and when the surface is inclined at a maximum angle then,

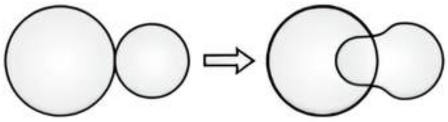
$$\delta m a \cos\theta = \delta m g \sin\theta$$

$$\frac{\sin\theta}{\cos\theta} = \frac{a}{g} \text{ is required slope.}$$

$$\tan\theta = \frac{a}{g} \text{ is required slope.}$$

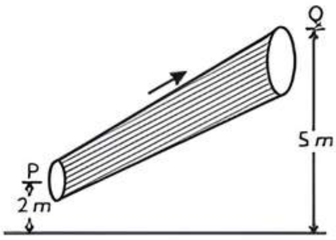
**26. Two droplets of water of varying diameters interacted with a portion of both bubbles. What will the shape of the common boundary look like when viewed from inside the relatively small sphere?**

**Ans.** When seen from inside the smaller droplet, the shape of the common boundary will appear concave as shown in the figure



For a curved liquid film, the pressure is greater on its concave side and pressure inside the smaller bubble is more than that inside the larger drop because $P \propto \frac{1}{r}$.

**27. A non-viscous liquid with a constant density of $10^3$ kg $m^{-2}$ flows in a streamlined motion along a variable cross-section tube. As shown in the figure, the tube is kept inclined in the vertical plane. The area of the cross-section of the tube at two points P and Q at heights of 2 m and 5 m is $4 \times 10^{-3}$ $m^2$ and $8 \times 10^{-3}$ $m^2$, respectively, and the velocity of the liquid at point P is 1 $ms^{-1}$.**




**Determine the work done per unit volume due to pressure and gravity forces as the liquid flows from point P to point Q.**

**Ans.** Let $a_1$ and $a_2$ be the areas of cross-section of the tube and $v_1$ and $v_2$ be the velocities of the liquid at the ends P and Q respectively. Further, suppose that $h_1$ and $h_2$ are heights of the ends P and Q above the horizontal, then

$$a_1 = 8 \times 10^{-3} m^2, a_2 = 16 \times 10^{-3} m^2$$

$$h_1 = 4m, h_2 = 5m, v_1 = 2 \text{ ms}^{-1}$$

Also, the density of the liquid.

According to the equation of continuity,

We have, $a_1 v_1 = a_2 v_2$

$$v_2 = \frac{a_1}{a_2} v_1 = \frac{16 \times 10^{-3}}{8 \times 10^{-3}} \times 2 = 4 \text{ms}^{-1}$$

If $P_1$ and $P_2$ are pressure at the ends P and Q, then according to Bernoulli's theorem,

$$\frac{P_1}{\rho} + gh_1 + \frac{1}{2} v_1^2 = \frac{P_2}{\rho} gh_2 + \frac{1}{2} v_2^2$$

Work done by the pressure forces per unit volume $= P_1 - P_2$

$$= \rho g(h_2 - h_1) + \frac{1}{2} \rho (v_2^2 - v_1^2)$$

$$= 10^3 \times 9.8(5-4) + \frac{1}{2} \times 10^3 \times (1^2 - 2^2)$$

$$= 9800 - 1500$$

$$= 8300$$

Work done by gravity forces per unit volume

$$= \rho g(h_2 - h_1)$$

$$= 10^3 \times 9.8(5-4) = 98 \text{ Jm}^{-3}$$

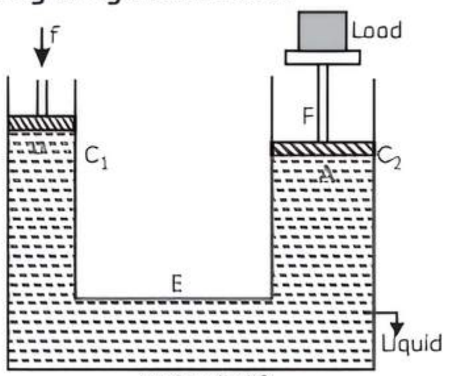
**28. Two vessels have the same base area but different shapes. The first vessel takes twice the volume of water that the second vessel requires to fill up to a particular common height. Is the force exerted by the water on the base of the vessel the same in the two cases? If so, why do the vessels filled with water to that same height give different readings on a weighing scale?**

**Ans.** Two vessels having the same base area have the identical force and equal pressure acting on their common base area. Since the shapes of the two vessels are different, the force exerted on the sides of the vessels has non-zero vertical components. When these vertical components are added, the total force on one vessel comes out to be greater than that on the other vessel. Hence, when these vessels are filled with water to the same height, they give different readings on a weighing scale.

**29. State the principle on which Hydraulic lift works and explain its working.**

**Ans.** Hydraulic lift works on the principle of Pascal's law. According to this law, in the absence of gravity, the pressure is the same at all points inside the liquid lying at the same horizontal plane.

**Working of Hydraulic effect:**




Hydraulic lift

$a$ = Area of cross-section of the piston at $C_1$

A = Area of cross-section of the piston at $C_2$.

Let a downward force $f$ be applied on piston $C_1$.

Then the pressure exerted on the liquid,

$$P = \frac{F}{A}$$

According to Pascal's law, this pressure is transmitted equally to the piston of cylinder D.

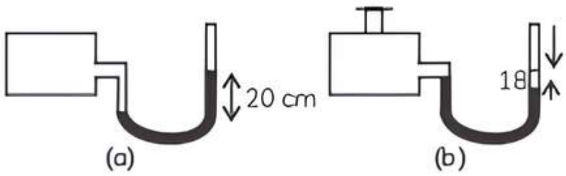
∴ Upward force acting on the piston of cylinder $C_2$ will be:

$$F = PA = \frac{F}{A}A$$

As A >> $a$, F >> $f$

*i.e.*, a small force applied on the smaller piston will be appearing as a very large force on the large piston. As a result this heavy load placed on a larger piston is easily lifted upwards.

**30. A manometer reads the pressure of a gas in an enclosure as shown in the fig. (a) when some of the gas is removed by a pump, the manometer reads as in fig (b). The liquid used in the manometer is mercury and the atmospheric pressure is 76 cm of mercury. Give absolute and gauge pressure of the gas in the enclosure for cases (a) and (b).**




**[Delhi Gov. QB 2022]**

**Ans.** For figure (a)

Atmospheric pressure, $P_0$ = 76 cm of Hg

Difference between the levels of mercury in the two limbs gives gauge pressure

Hence, gauge pressure is 20 cm of Hg

Absolute pressure = Atmospheric pressure + Gauge pressure

= 76 + 20 = 96 cm of Hg

For figure (b)

Difference between the levels of mercury in the two limbs = – 18 cm

Hence, the gauge pressure is –18 cm of Hg.

Absolute pressure = Atmospheric pressure + Gauge pressure

= 76 cm – 18 cm = 58 cm

**31. A piece of ice, with a stone frozen inside it, floats on water taken in a beaker. Will the level of water increase or decrease or remain the same when the ice melts completely? [Diksha]**

**Ans.** Let $M$ be the mass of an ice piece and $m$ that of stone. As this combination of mass (M + $m$) floats in water, the mass of water displaced is (M + $m$).

If ρ is the density of water, then the volume of water is displaced is,

$$V = \frac{M+m}{\rho}$$

When ice melts, we get extra water of mass M and volume $M\rho$. The stone sinks and displaces water equal to its own volume $\frac{m}{d}$, where $d$ is the density of the stone.

Thus, total the volume of extra water (obtained by melting of ice) and water displaced by stone is,

$$V' = \frac{M}{\rho} + \frac{m}{d}$$

## LONG ANSWER Type Questions (LA)

[ **4** & **5** marks ]

**32. Discuss the variation of fluid pressure with depth. Hence explain how Pascal's law is affected in the presence of gravity. [Diksha]**

**Ans. Variation of liquid pressure with depth:** Consider a liquid at rest in a container. The liquid pressure must be the same at all points which are at the same depth, as otherwise liquid will not be in equilibrium.

**Pascal's law is modified in the presence of gravity:**

Consider a vessel containing a fluid. As the fluid is in equilibrium, so every volume element of the fluid is also in equilibrium. Consider one volume element in the form of a cylindrical column of fluid of height $h$ and area of cross-section $A$.

The various forces acting on the cylindrical column of the fluid are:

(1) Force, $F_1 = P_1A$ acting vertically downward on the top face of the column $P_1$ is the pressure of the fluid on the top face of the column.

(2) Force, $F_2 = P_2A$ acting vertically upward at the bottom face of the cylindrical column. $P_2$ is the pressure of the liquid on the bottom face of the column.

(3) Weight, $W = mg$ of the cylindrical column of the fluid acting vertically downward Since, the cylindrical column of the fluid is in equilibrium, so the net force acting on the column is zero.

i.e., $F_1 + W - F_2 = 0$

$$P_1A + mg - P_2A = 0$$

$$P_1A + mg = P_2A$$

$$P_2 = P_1 + \frac{M}{A}g \quad \text{... (i)}$$

Now, the mass of the cylindrical column of the fluid is, $m$ = volume × density of the fluid

= Area of cross section × height × density

= $Ah\rho$

Therefore, eqn. (i) becomes,

$$P_2 = P_1 + Ah\rho g$$

And $P_2 = P_1 + h\rho g$

**33. (A) Pressure decreases as one ascends the atmosphere. If the density of air is $r$. What is the change in pressure $dP$ over a differential height $dh$?**

**(B) Consider the pressure P to be proportional to the density, find the pressure P at a height $h$ if the pressure on the surface of the earth is $P_0$.**

**(C) This model of the atmosphere works for relatively small distances. Identify the underlying assumption that limits the model. [NCERT Exemplar]**

**Ans.** (A) Consider a part (packet) of the atmosphere of thickness $dh$. As the pressure at a point in a fluid is equal in all directions. So, the pressure on the upper layer is P downward and on the lower layer is (P + dP) upward. Force due to pressure is balanced by Buoyant force by air.

$$(P + dP)A - P.A = -V\rho g$$

$$P.A + dP\,A - P.A = -A\,dh\,\rho g$$

$$dPA = -\rho g dh A$$

$$dP = -\rho g dh$$

Negative sign shows that pressure decreases as height increases.

(B) Let $\rho_0$ is the density of air on the surface of the earth.

Pressure P at a point is directly proportional to density.

$$P \propto \rho$$

or $$\frac{P}{P_o} = \frac{\rho}{\rho_o}, \rho = \left(\frac{P}{P_0}\right)$$

$$dP = -\rho g dh$$

($\rho$ is the density of air in the atmosphere)

$$dP = -\left(\frac{P}{P_0}\right)\rho_0 g dh$$

$$\frac{dP}{\rho} = \frac{-\rho_0 g}{P_0}dh$$

Integrating both sides

$$\int_{P_o}^{n} \log P dP = -\int_0^h \frac{\rho_0 g}{P_0}dh$$

$$\log\left(\frac{P}{P_0}\right) = \frac{-\rho_0}{P_0}gh$$

$$\frac{P}{P_0} = e^{\rho_o gh/P_o}$$

$$P = P_0 e^{\rho_o gh/P_o}$$

(C) Temperature (T) remains constant only near the surface of the earth, not at a greater height.

## NUMERICAL Type Questions

**34. The sap in trees, which consists mainly of water in summer, rises in a system of capillaries of radius, $r = 2.5 \times 10^{-5}$m. The surface tension of sap is $T = 7.28 \times 10^{-2}$ $Nm^{-1}$ and the angle of contact is 0°. Does the surface tension alone account for the supply of water to the top of all trees?**

**[NCERT Exemplar](2m)**

**Ans.** Capillarity $r = 2.5 \times 10^{-5}$m

S.T. $= 7.28 \times 10^{-2}\ Nm^{-1}$

$g = 9.8\ m/s^2$

$\theta = 0°, \rho = 10^3\ kg/m^3$

$$h = \frac{2S\cos\theta}{r\rho g}$$

$$= \frac{2 \times 7.28 \times 10^{-2}\cos 0°}{2.5 \times 10^{-5} \times 10^3 \times 9.8}$$

$$= \frac{104}{175} \times \frac{10^3}{10^3}$$

$$= 0.597 = 0.6 \text{ m}$$

**35. The manual of a car instructs the owner to inflate the tyres to a pressure of 200 k pa.**

**(A) What is the recommended gauge pressure?**

**(B) What is the recommended absolute pressure?**

(C) If, after the required inflation of the tyres, the car is driven to a mountain peak where the atmospheric pressure is 10% below that at sea level, what will the tyre gauge read?

[Delhi Gov. QB 2022](3m)

**Ans.** (A) Pressure instructed by the manual

$= P_g = 200 \text{ k } P_a$

(B) Absolute Pressure

$= 101 \text{ k } P_a + 200 \text{ k } P_a$

$= 301 \text{ k } P_a$

(C) At mountain Peak $P_a'$ is 10% less

$P_a' = 90 \text{ k } P_a$

If we assume absolute pressure in tyre does not change during driving then

$P_g = P - P_a' = 301 - 90$

$= 211 \text{ k } P_a$

So, the tyre will read 211 k $P_a$ pressure.

**36.** **The river goes at a rate of 10 liters per second through an orifice at the river's bottom, which is 1000 cm deep. Determine the rate of water evaporation if an additional pressure of 40 kg/cm² is applied to the water's surface. (3m)**

**Ans.** Given: $h = 1000$ cm.

$v = \sqrt{2gh} = \sqrt{2 \times 980 \times 100}$ m/s

$= 1400$ cm/s;

$v = 10$ liters/s

Additional pressure

$= 40 \text{ kg/cm}^2$

$= 40000 \text{ g/cm}^2$

Now, $40000 \text{ g/cm}^2$

$= \dfrac{40000 \times 980}{980}$ cm of water

$= 40000$ cm of water

Pressure head, $h_1 = 40000 + 1000 = 41000$ cm

New velocity, $v_1 = \sqrt{2gh_1}$

$= \sqrt{2 \times 980 \times 41000}$ cm/s

$v = av$ and $v_1 = av_1$

$v_1 = v\dfrac{v_1}{v} = 10\sqrt{\dfrac{41000}{1000}}$

$= 64.03$ l/s

**37.** **The terminal velocity of a tiny droplet is $v$. N number of such identical droplets combine together forming a bigger drop. Find the terminal velocity of the bigger drop.**

**[Delhi Gov. QB 2022] (3m)**

**Ans.**

$$v = \frac{2}{9}\left[\frac{g(\sigma - \rho)r^2}{\eta}\right]$$

$$\frac{v}{r^2} = \frac{2g}{9\eta}(\sigma - \rho) \quad \text{—(i)}$$

Similarly, $\dfrac{v'}{R^2} = \dfrac{2g}{9\eta}(\sigma - \rho)$ —(ii)

Dividing (i) by (ii)

$$\frac{v}{v'} = \frac{r^2}{R^2}$$

$$\Rightarrow \quad v' = v\left(\frac{R}{r}\right)^2$$

If N drops coalesce, then

Volume of one big drop = Volume of N droplets

$$\frac{4}{3}\pi R^3 = N\left(\frac{4}{3}\pi r^3\right)$$

$$R = N^{\frac{1}{3}} r$$

∴ Terminal velocity of bigger drop

$= \left(\dfrac{R}{r}\right)^2 \times v$ from equation (i)

$= N^{2/3}\, v$ from equation (ii)

**38.** **When an air pocket rises from the bottom of a lake to its surface, its volume increases by 10 times. What is the depth of the lake if the barometric height is 0.96 m of Hg (density of Hg is 13.5 g/cm³ and $g = 9.8$ m/s²)? (5m)**

**Ans.** Let $P_1$ $P_2$ and $V_1$ $V_2$ are pressure and volume at the bottom and top respectively.

Since temp remaining constant

$P_1V_1 = P_2V_2$

Given, $V_2 = 10V_1$

$\Rightarrow$ $P_1 = 10P_2$

Hence, the difference in pressure at the top and bottom:

$P_1 - P_2 = 9P_2$

Given density of Hg $= 13.5$ g/cm³

$= 13500$ kg/m³

Height of mercury column $= 0.96$ m

Acceleration due to gravity $= 9.8$ m/s²

Hence, $P_2 = \rho g h$

$= 3500 \times 9.8 \times 0.96$

$= 127008$

$\Rightarrow$ Pressure difference $= 9P_2$

$= 9 \times 127008$

$= 1,143,072$

This pressure is equivalent to a water pressure of height $h$.

Hence, $\rho g h = 1,143,072$

$\Rightarrow$ $1000 \times 9.8 \times h = 1,143,072$

$\Rightarrow$ $h = 116.64$ m

Hence the height of the lake will be 116.64 m.

# VISCOSITY AND SURFACE TENSION 2

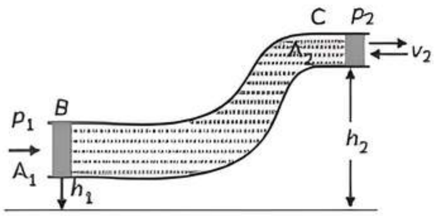
## TOPIC 1

## BERNOULLI'S THEOREM AND ITS APPLICATION

### Bernoulli's Theorem

According to Bernoulli's principle, "The sum of pressure energy, kinetic energy and potential energy per unit volume of an incompressible, non-viscous fluid in a streamlined irrational flow remains constant along a streamline"

$$p + \rho g h + \frac{1}{2}\rho v^2 = \text{constant}$$




Bernoulli's equation is valid only for incompressible steady flow of a fluid with no viscosity.

### Applications of Bernoulli's Theorem

#### Velocity of Efflux (Torricelli's Theorem)

Let us find the velocity with which liquid comes out of a hole at a depth $h$ below the liquid's surface.

Using Bernoulli's Theorem

$$P_A + \frac{1}{2}\rho v_A^2 + \rho g h_A = P_B + \frac{1}{2}\rho v_B^2 + \rho g h_B$$

$$P_{atm} + \frac{1}{2}\rho v_A^2 + \rho g h_A = P_{atm} + \frac{1}{2}\rho v_B^2 + 0$$

Note: $P_B = P_{atm}$

Because we have opened the liquid to the atmosphere.

$$v^2 = v_A^2 + 2gh$$

Using the equation of continuity,

$$Av_A = a_v$$

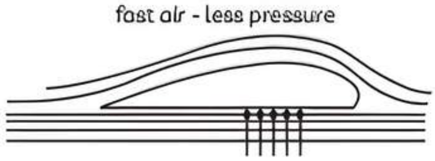
Where A=cross section of vessel, $a$=area of hole

$$v^2 = \frac{a^2}{A^2}v^2 + 2gh$$

$$v^2 = \frac{\sqrt{2gh}}{\sqrt{\frac{1-a^2}{A^2}}} \approx 2gh$$

#### Dynamic Lift

**Wings of Aircraft:** The tapering of the aircraft's wings is depicted in the figure. Because of the specific shape of the wings, when the aircraft runs, air passes over it at a faster rate than over its lower surface. According to Bernoulli's principle, the difference in airspeeds above and below the wings creates a pressure difference, which causes an upward force known as 'dynamic lift' to act on the plane. If this force becomes greater than the plane's weight, the plane will rise.




**Ball moving without spin:** At corresponding points, the velocity of the fluid (air) above and below the ball is the same, resulting in a zero-pressure difference. As a result, the air exerts no upward or downward force on the ball.

**Ball moving with spin:** A spinning ball drags air along with it. More air will be dragged if the surface is rough. The ball is moving forward, while the air is moving backwards in relation to it. As a result, the velocity of air above the ball is greater than that of air below it. As a result, the streamlines become crowded above and scarce below.

The difference in air velocities causes a pressure difference between the lower and upper faces, resulting in a net upward force on the ball. The Magnus effect refers to the dynamic lift caused by spinning.

**Example 2.1: Can Bernoulli's equation be used to describe the flow of water through a rapid in a river? [NCERT]**

**Ans.** No, Bernoulli's equation cannot be used to describe the rapid flow of water in a river because the rapid flow is turbulent and not streamlined. Bernoulli's equation is applicable to streamline the flow of liquid.

**Example 2.2:** Does it matter if one uses gauge instead of absolute pressures in applying Bernoulli's equation? Explain [NCERT]

**Ans.** No, it does not matter if one uses gauge pressure instead of absolute pressures in applying Bernoulli's equation, provided the atmospheric pressure at the two points where Bernoulli's equation is applied are significantly different.

**Example 2.3:** Water is maintained at a height of 10 m in a tank. Calculate the diameter of a circular aperture needed at the base of the tank to discharge water at the rate of 26.4 $m^3sec^{-1}$. Given that $g = 9.8\ ms^{-2}$ [NCERT]

**Ans.** Here, $h = 10$ m, $g = 9.8\ ms^{-2}$

Velocity of water from the bottom aperture

$$v = \sqrt{2gh}$$

$$= \sqrt{2 \times 9.8 \times 10}$$

$$= 14\ ms^{-1}$$

If $a$ is an area of cross-section of the circular aperture,

then the rate of discharge of the liquid,

$$V = av = \frac{\pi D^2}{4} \times v$$

Here, $V = 26.4\ m^3\ sec^{-1}$

$$= \frac{26.4}{60} = 0.44\ m^3s^{-1}$$

$$0.44 = \frac{\pi D^2}{4} \times 14$$

$$D^2 = \frac{0.44 \times 4}{\pi \times 14} = 0.2\ m$$

# TOPIC 2

# VISCOSITY

The property of a fluid by which it opposes the relative motion between its different layers is known as viscosity and the force that is into play is called the viscous force.

Viscous force is given by:

$$F = -\eta A \frac{dv}{dx}$$

Where $\eta$ is a constant depending upon the nature of the liquid and is called the coefficient of viscosity

and velocity gradient $= \frac{dv}{dx}$

S.I Unit of coefficient of viscosity is pa.s or $Nsm^{-2}$.

CGS unit of viscosity is poise. (1 Pa.s = 10 Poise)

## Critical Velocity

The critical velocity is the velocity of liquid flow, up to which, its flow is streamlined and above which its flow becomes turbulent.

It is given by $v_c = \frac{k\eta}{\rho r}$

The critical velocity of a liquid depends on the coefficient of viscosity of the liquid, density of the liquid and radius of the tube.

## Stoke's Law

When a solid move through a viscous medium, its motion is opposed by a viscous force that varies according to the velocity, shape, and size of the body. The viscous drag on a spherical body of radius $r$ moving at velocity v in a viscous medium is given by

$$F_{Viscous} = 6\pi\rho r v$$

This relation is called Stoke's law.

This law is used in the determination of electronic charges with the help of Millikan's experiment. This law explains why the speed of raindrops is less than that of a body falling freely with a constant velocity from the height of clouds.

## Terminal Velocity

It is the maximum constant velocity acquired by the body while falling freely in a viscous medium.

$$v_r = \frac{2r^2(\rho - \rho_0)g}{9\eta}$$

## Poiseuille's Formula

Poiseuille studied the stream-line flow of liquid in capillary tubes.

Volume of liquid coming out of tube per second is given by. $v_l = \frac{\pi p r^4}{8\eta l}$

## Reynold's Number

Osborne Reynolds defined a dimensionless parameter whose value decides the nature of the flow of a liquid through a pipe, *i.e.*, whether a flow will be steady or turbulent.

Reynold's number, $R_e = \frac{\rho v D}{\eta}$

The exact value at which turbulence sets in a fluid is called critical Reynold's number.

**Example 2.4:** A steel plate 100 $cm^2$ in size rests on a 0.2 cm thick layer of castor oil ($\eta \simeq 15.5$ poise). Determine the horizontal force required to move the plate at 3 cm/s. [NCERT]

**Ans.** As, $F = -\eta A \frac{dv}{dx}$

Where, $\eta$ = 15.5 poise, A = 100 $cm^2$

$$\frac{dv}{dx} = \frac{3}{0.2} = 15 s^{-1}$$

$$F = 15.5 \times 100 \times 15$$
$$= -23250 \text{ dyne}$$
$$= -0.233 \text{ N}$$

**Example 2.5:**

**(A) What is the largest average blood flow velocity in an artery with a radius of $2 \times 10^{-3}$ m if the flow must remain laminar?**

**(B) What is the corresponding flow rate? Take the coefficient of viscosity of blood to be $2.084 \times 10^3$ pa s. [NCERT]**

**Ans.** (A) The maximum value of Reynold's number for laminar flow is 2000. Hence, the critical velocity is,

$$v_c = \frac{R_e \eta}{\rho D} = \frac{2000 \times 2.084 \times 10^{3}}{1.06 \times 10^3 \times 4 \times 10^{3}} = 0.98 \text{ m/s}$$

(B) Volume of blood flowing per second.

$$V = Av_c$$
$$= \pi r^2 v_c$$
$$= \frac{22}{7} \times (2 \times 10^{-3})^2 \times 9.8$$
$$= 1.23 \times 10^{-5} \text{ m}^3$$

# TOPIC 3

# SURFACE TENSION

It is the property of a liquid at rest that causes the free surface of the liquid to have a small area. It is expressed as the force experienced per unit length of an imaginary line drawn on the liquid's surface.

So, $S = \frac{F}{l}$

Where, S = surface tension of liquid.

At a constant temperature, it is also defined as the work done in increasing the area of a liquid surface by one against the force of surface tension.
It has the SI unit of N/m.

The surface tension of a liquid is defined as the force per unit length in the plane of the liquid surface at right angles to either side of an imaginary line drawn on that surface. The unit of surface tension in the MKS system is N/m, J/$m^2$, as per the CGS system, it is Dyne/cm, erg/$cm^2$.

## Applications of Surface Tension

(1) Raindrops are spherical in shape.

(2) The hair of a shaving brush clings together when taken out of water.

(3) Oil spreads in cold water but remains as a drop in hot water, etc.

## Surface Energy

According to the molecular theory of surface tension, molecules at the surface have some extra energy as a result of their position. Surface energy is the additional energy per unit area of the surface.

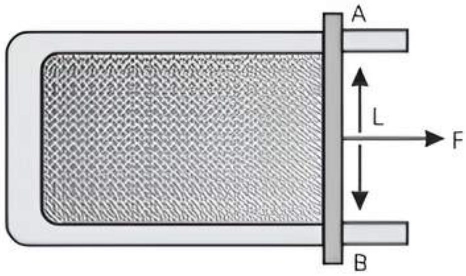
Surface energy

$$= \frac{\text{work done in increasing the surface area}}{\text{Increase in surface area}}$$

Its SI unit is $Jm^{-2}$.

## Relation Between Surface Energy and Surface Tension

The work must be done over the surface of the liquid in order to increase the surface area. This work is stored as potential energy in the liquid surface. As a result, a liquid's surface energy can be defined as the excess potential energy per unit area of the liquid's surface.




$$W = S\Delta A$$

Where, $\Delta A$ = increase in surface area.

## Angle of Contact

The angle of contact is defined as the angle formed by the tangent to the liquid surface at the point of contact and the solid surface within the liquid.

Examples:

When a glass plate is immersed in mercury, the surface curves and the mercury sink below. Mercury has an obtuse angle of contact.

When the plate is immersed in water with one of its sides vertically, the water is drawn up along the plane and takes on the curved shape shown. Water has an acute angle of contact.

## Drops and Bubbles

Excess pressure in a liquid drop or bubble in a liquid is

$$P = \frac{2T}{R}$$

Excess pressure in a soap bubble is

$$P = \frac{4T}{R}$$

(because it has two free surfaces)

Excess pressure inside an air bubble inside a liquid

$$P = \frac{2T}{R}$$

**Example 2.6: Evaluate the work done against surface tension in blowing a soap bubble from a radius of 10 cm to a radius of 20 cm if the surface tension of the soap solution is 25 × $10^{-3}$ N/m. Then compare it to a liquid drop with the same radius.** **[NCERT]**

**Ans.** (A) **For soap bubble:** Extension in the area:

Work done, $W_1$

= surface tension × extension in area

$= 25 \times 10^{-3} \times 0.24\pi = 6\pi \times 10^{-3}$ J

(B) **For liquid drop:** In case of the liquid drop only one free surface, so the extension in the area will be half of the soap bubble.

$$W_2 = \frac{W_1}{2} = 3\pi \times 10^{-3} J$$

**Example 2.7: In Millikan's oil drop experiment, what is the terminal speed of an uncharged drop of radius 2.0 × $10^{-5}$ m and density 1.2 × $10^3$ kg $m^{-3}$. Take the viscosity of air at the temperature of the experiment to be 1.8 × $10^{-5}$ Pa s. How much is the viscous force on the drop at that speed? Neglect buoyancy of the drop due to air.** **[NCERT]**

**Ans.** Terminal speed = 5.8 cm/s;

Viscous force = 3.9 × $10^{-10}$ N

Radius of the given uncharged drop,

$r = 2.0 \times 10^{-5}$ m

Density of the uncharged drop,

$\rho = 1.2 \times 10^3$ kg $m^{-3}$

Viscosity of air, $\eta = 1.8 \times 10^{-5}$ Pa s

Density of air ($\rho_o$) can be taken as zero to neglect the buoyancy of air.

Acceleration due to gravity, $g = 9.8$ m/s$^2$

Terminal velocity ($v$) is given by the relation,

$$v = \frac{2r^2 \times (\rho - \rho_o)}{9\eta}$$

$$= \frac{2 \times (2.0 \times 10^{-5})^2 (1.2 \times 10^3 - 0)}{9 \times 1.8 \times 10^{-5}}$$

$= 5.807 \times 10^{-2}$ ms$^{-1}$

$= 5.8$ cms$^{-1}$

Hence, the terminal speed of the drop is 5.8 cm s$^{-1}$.

The viscous force on the drop is given by:

$F = 6\pi\eta rv$

$F = 6 \times 3.14 \times 1.8 \times 10^{-5} \times 2.0 \times 10^{-5} \times 5.8 \times 10^{-2}$

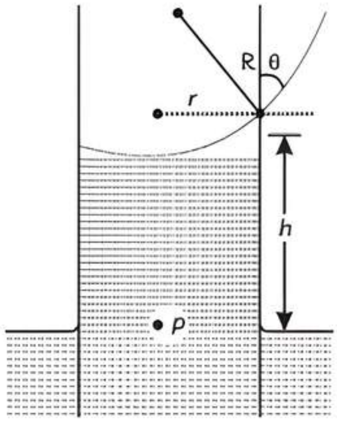
$F = 3.9 \times 10^{-10}$ N

Hence the viscous force on the drop is,

$= 3.9 \times 10^{-10}$ N

## Capillary Tube and Capillarity Action

A capillary tube is a very narrow glass tube with a fine bore that is open at both ends. Capillarity occurs when a capillary tube is dipped in a liquid and the liquid rises or falls in the tube. Capillarity is the name given to this action.




$$h = \frac{2S\cos\theta}{r\rho g}$$

Where, S = surface tension,

θ = angle of contact,

$r$ = radius of a capillary tube,

R = radius of a meniscus, and

ρ = density of a liquid.

## Capillary Rise in a Tube of Insufficient Length

If the actual height to which a liquid will rise in a capillary tube is '$h$' then a capillary tube of length less than '$h$' can be called a tube of "insufficient length".

In such a case, the liquid rises to the top of the capillary tube of length $l$ ($l < h$) and adjusts the radius of curvature of its meniscus until the excess pressure is equalized by the pressure of the liquid column of length $l$ (Note: liquid does not overflow).

$$\frac{2\sigma}{r'} = l\rho g$$

If $r$ is the actual radius of curvature,

$$\frac{2\sigma}{r} = l\rho g 1$$

Comparing both equations,

$$\frac{2\sigma}{\rho g} = lr' = hr$$

$$\frac{hr}{l} = r'$$

i.e., the radius of curvature $r'$ can be calculated

$$h = l\cos\alpha$$

$$\Rightarrow \quad l = \frac{h}{\cos\alpha}$$

**Example 2.8:**

Assertion (A): The water rises higher in a capillary table of small diameter than in the capillary tube of large diameter.

Reason (R): Height through which liquid rises in a capillary tube is inversely proportional to the diameter of the capillary tube.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true and R is not correct explanation of A.

(c) A is true but R is false.

(d) A is false and R is also false.

**Ans.** (a) *Both A and R are true and R is the correct explanation of A.*

**Explanation:** The height of capillary rise is inversely proportional to radius of capillary tube.

i.e. $h \propto \frac{1}{r}$ So for smaller '$r$' the value of '$h$' is higher.

**Example 2.9:** Mercury has an angle of contact equal to 140° with soda lime glass. A narrow tube of radius 1.00 mm made of this glass is dipped in a trough containing mercury. By what amount does the mercury dip down in the tube relative to the liquid surface outside?

Surface tension of mercury at the temperature of the experiment is 0.465 N $m^{-1}$.

Density of mercury = 13.6 × $10^3$ kg $m^{-3}$. [NCERT]

**Ans.** Angle of contact, θ = 140°

Radius of the tube, $r$ = 1 mm = $10^{-3}$m

Surface tension of mercury S = 0.465 N/m

Density of mercury, ρ = 13.6 × $10^3$ kg/$m^3$

$g = 9.8$ m/$s^2$

Let the dip in height of mercury be $h$.

Surface tension is given by,

$$S = P_0 e^{-y/y_0}$$

$$h = \frac{2S\cos\theta}{\rho g r}$$

$$h = \frac{2 \times 0.465 \times \cos 140^\circ}{13.6 \times 10^3 \times 9.8 \times 10^{-3}}$$

$$h = -5.34 \text{ mm}$$

Negative sign indicates that the mercury level dips by 5.34 mm.

**Example 2.10:** Two narrow bores of diameters 3.0 mm and 6.0 mm are joined together to form a U-tube open at both ends. If the U-tube contains water, what is the difference in its levels in the two limbs of the tube? Surface tension of water at the temperature of the experiment is 7.3 × $10^{-2}$ N$m^{-1}$. (Take the angle of contact to be zero and the density of water to be 1.0 × $10^3$ kg $m^{-3}$ ($g$ = 9.8 $ms^{-2}$) [NCERT]

**Ans.** Given that:

When the angle of contact is zero degree the radius of the meniscus equals the radius of bore.

Excess pressure in the first bore:

$$P_2 = \frac{2T}{r_2}$$

$$= \frac{2 \times 7.3 \times 10^{-2}}{3 \times 10^{-3}}$$

$$= 48.7 \text{ Pascal}$$

Excess pressure in the second bore,

$$P_1 = \frac{2T}{r_2}$$

$$= \frac{2 \times 7.3 \times 10^{-2}}{1.5 \times 10^{-3}}$$

$$= 97.3 \text{ Pascal}$$

Hence, the pressure difference in the two limbs of the tube,

$$\Delta P = P_1 - P_2 = h\rho g$$

Or $$h = \frac{P_1 - P_2}{\rho g}$$

$$= \frac{97.3 - 48.7}{1.0 \times 10^3 \times 9.8} = 5.0 \text{ mm}$$

## OBJECTIVE Type Questions

[ **1** mark ]

### Multiple Choice Questions

**1.** While washing dirty clothes lots of detergents is kept inside a wash tub for remove the dirt or greasy strains this is done due to many reasons, some are stated below. Check each statement and find which is correct.

(I) It raises the oil-water surface tension.

(II) It reduces the surface tension of oil in water.

(III) It causes the solution's viscosity to rise.

(IV) Dirt is held suspended by detergent molecules.

Options:

(a) (I) and (II) (b) (I) only

(c) (III) and (IV) (d) (IV) only

**Ans.** *(a) (I) and (II)*

**Explanation:** Adding detergents to water helps in removing dirty greasy stains. This is because it decreases the oil-water surface tension and dirt is held suspended surrounded by detergent molecules.

**2.** **A soap bubble in a vacuum has a radius of 3 cm and another soap bubble in a vacuum has a radius of 4 cm. If the two bubbles coalesce under isothermal conditions, then the radius of the new bubble is:**

(a) 2.3 cm (b) 4.5 cm

(c) 5 cm (d) 7 cm

**Ans.** *(c) 5 cm*

**Explanation:** When two bubbles coalesce than the total number of molecules of air will remain the same and the temperature will also remain constant.

So, $$n_1 + n_2 = n$$

$$P_1V_1 + P_2V_2 = PV$$

$$\frac{4T}{r_1}\left(\frac{4}{3}\pi r_1^3\right) + \frac{4T}{r_2}\left(\frac{4}{3}\pi r_2^3\right) = \frac{4T}{r_1}\left(\frac{4}{3}\pi r^3\right)$$

$$r = \sqrt{r_1^2 + r_2^2}$$

So, $$r = \sqrt{3^3 + 4^2} = 5 \text{ cm}$$

### Related Theory

→ *If a bubble is formed inside a bubble then inside pressure is more than the pressure outside the bubble.*

**3.** **Spiders and insects move and run about on the surface of the water without sinking because.**

**(a) Elastic membrane is formed on water due to the property of surface tension.**

**(b) Spiders and insects are lighter**

**(c) Spiders and insects swim on the water**

**(d) Spiders and insects experience up-thrust**

**Ans.** *(a) Elastic membrane is formed on water due to the property of surface tension.*

**Explanation:** Spiders and insects move and run about on the surface of the water without sinking because the weight of spiders or insects can be balanced by the vertical component of force due to surface tension.

**4.** **The angle of contact at the interface of water glass is 0°, Ethyl alcohol–glass is 0°, Mercury–glass is 140° and Methyl iodide-glass is 30°. A glass capillary is put in a trough containing one of these four liquids. It is observed that the meniscus is convex. The liquid in the trough is:**

(a) water (b) ethyl alcohol

(c) mercury (d) methyl iodide

[Delhi Gov. QB 2022]

**Ans.** *(c) mercury*

**Explanation:** If the angle of contact is obtuse, the liquid meniscus in a capillary tube will be convex upwards. This happens when one end of a glass capillary tube is immersed in a trough of mercury.

**5.** **In a capillary tube experiment, a vertical 30 cm long capillary tube is dipped in water. The water rises to a height of 10 cm due to capillary action. If this experiment is conducted in a freely falling elevator, the length of the water column becomes:**

(a) 10 cm (b) 20 cm

(c) 30 cm (d) Zero

**Ans.** *(c) 30 cm*

**Explanation:** Inside a freely falling elevator, water will rise up to the top level but still not overflow. Radius of curvature (R') increases in such a way that the final height *h'* is reduced and given by

$$h' = \frac{hR}{R'}$$

(It is in accordance with Zurin's law).

Height of liquid in a capillary

$$T = \frac{rhdg}{2}$$

$$h = \frac{2T}{rdg}$$

In a freely falling elevator, gravitational acceleration will be zero

$$\therefore \quad h = \infty.$$

i.e., the water will rise to the maximum available height. Hence, 30 cm.

### Caution

→ *Students must know that if a capillary tube is dipped into a liquid and tilted at some angle α from vertical then the vertical height of the liquid column remains same whereas the length of the liquid column in the capillary tube increases.*

**6.** **The knowledge of different liquids and their variation in temperature helps us to select suitable lubricants for the machine.**

**(a) Surface Tension**

**(b) Coefficient of Viscosity**

**(c) Coefficient of Linear expansion**

**(d) None of these** **[Diksha]**

**Ans.** *(b) Coefficient of Viscosity*

**Explanation:** As different kinds of liquids are used in machines to avoid friction, therefore knowledge of the coefficient of viscosity of those liquids becomes important to avoid the wearing and tearing of machinery parts.

7. A person is standing near a railway track when a fast-moving train passes close to him.
(a) He will be pulled towards the train because a fast-moving train decreases the pressure between the person and the train.
(b) He will be pulled towards the train because a fast-moving train increases the pressure between the person and the train.
(c) He will be pushed away by the train because a fast-moving train decreases the pressure between the person and the train.
(d) He will be pushed away by the train because a fast-moving train increases the pressure between the person and the train.

**Ans.** (a) *He will be pulled towards the train because a fast-moving train decreases the pressure between the person and the train.*

**Explanation:** Air near the fast-moving train also starts moving at high speed. According to Bernoulli's principle, it will decrease with pressure near the train. Hence, with a greater value of pressure on the other side of the person, the person gets pulled towards the train.

8. If the radius of two different soap bubbles is $R_1$ and $R_2$ respectively combined in vacuum in isothermal conditions to form a single soap bubble then the radius of the combined soap bubble is:

(a) $\frac{R_1R_2}{R_1+R_2}$ (b) $\sqrt{R_1^2+R_2^2}$

(c) $\frac{R_1R_2}{2}$ (d) $\frac{R_1R_2}{R_1 \quad R_2}$

**Ans.** (b) $\sqrt{R_1^2+R_2^2}$

**Explanation:** When two bubbles coalesce then the total number of molecules of air will remain the same and the temperature also remains constant.

So, $$n_1 + n_2 = n$$

$$P_1V_1 + P_2V_2 = PV$$

$$\frac{4T}{R_1}\left(\frac{4}{3}\pi R_1^3\right)+\frac{4T}{R_1}\left(\frac{4}{3}\pi R_2^3\right)=\frac{4T}{R_1}\left(\frac{4}{3}\pi R^3\right)$$

$$R=\sqrt{R_1^2+R_2^2}$$

9. The radii of two drops falling are in the ratio 4 : 5. Their terminal velocities are in the ratio of:
(a) 16 : 25 (b) 4 : 5
(c) 25 : 16 (d) 5 : 4 [Diksha]

**Ans.** (a) *16 : 25*

**Explanation:**

As $$\frac{v_1}{v_2} = \left(\frac{R_1}{R_2}\right)^2$$

$$\frac{v_1}{v_2} = \left(\frac{4}{5}\right)^2$$

$$= \frac{16}{25}$$

10. The maximum height to which the water can be filled without leakage a vessel, whose bottom has round holes with a diameter of 0.1 mm is filled with water, is:
(a) 100 cm (b) 75 cm
(c) 50 cm (d) 30 cm

**Ans.** (d) *30 cm*

**Explanation:**

As, $$\frac{2T}{r} = h\rho g$$

$$h = \frac{2T}{r}$$

$$= \frac{2\times75}{\left(\frac{0.1}{2}\times10^{-1}\right)\times1\times1000}$$

$$= 30 \text{ cm}$$

## Assertion Reason Questions

Two statements are given one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below:
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true and R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false and R is also false.

11. Assertion (A): Where pressure is high, the velocity of the horizontal flow of an ideal liquid is lower and vice versa.
Reason (R): The total energy per unit mass remains constant in the streamlined flow of an ideal liquid, according to Bernoulli's theorem.

**Ans.** (a) *Both A and R are true and R is the correct explanation of A.*

**Explanation:** According to Bernoulli's theorem, for the streamlined flow of an ideal liquid, the total energy per unit mass is constant so if pressure is increased then the velocity of the flow has to decrease.

That is, $$P+\frac{\rho V^2}{2}+\rho gh=C$$

**Caution**

➥ *Students must know that practically some energy of the fluid gets converted into heat energy and is lost. But Bernoulli's equation is derived without considering this loss of energy.*

**12.** **Assertion (A):** The velocity of the flow of a liquid is smaller when pressure is larger and vice versa.

**Reason (R):** According to Bernoulli's theorem for the streamline flow of an ideal fluid, the total energy per unit mass remains constant.

**[Delhi Gov. QB 2022]**

**Ans.** *(d) A is false and R is also false.*

**Explanation:** According to Bernoulli's theorem, "The total energy per unit mass in the stream line flow of an ideal liquid is constant."

$$\text{That is } P+\frac{\rho v^2}{2}+\rho gh=C$$

The velocity of a liquid is determined by its pressure as well as the potential height available.

**13.** **Assertion (A):** With increasing temperature, the angle of contact of a liquid decreases.

**Reason (R):** The surface tension of a liquid increases as its temperature rises.

**Ans.** *(c) A is true but R is false.*

**Explanation:** The surface tension of liquid decreases with the increase in temperature.

As the temperature of the liquid increases, the surface tension and viscosity of the liquid decrease, which decreases the fluid tendency to hold molecules at a higher angle of contact.

**14.** **Assertion (A):** Soil particles inside water are freely placed but they stick together when taken out of water.

**Reason (R):** Thin films formed create pressure differences.

**Ans.** *(a) Both A and R are true and R is the correct explanation of A.*

**Explanation:** When taken out between the molecules a thin layer of the liquid film is formed and due to the contraction nature of the liquid surface, they come closer. When we take out soil from the river, we notice that all the particles stick together whereas when we look at the soil at the bottom of the river, we notice that they do not stick together.

This happens mainly because of the phenomenon of surface tension.

**15.** **Assertion (A):** Under the steady flow, the velocity of the particle of fluid is not constant at a point.

**Reason (R):** Ideal fluids are compressible.

**Ans.** *(d) A is false and R is also false.*

**Explanation:** In a steady fluid flow, the velocity of all particles at a point is the same. The path taken by a fluid particle under a steady flow is streamline. It is defined as a curve whose tangent at any point is in the direction of the fluid velocity at that point.

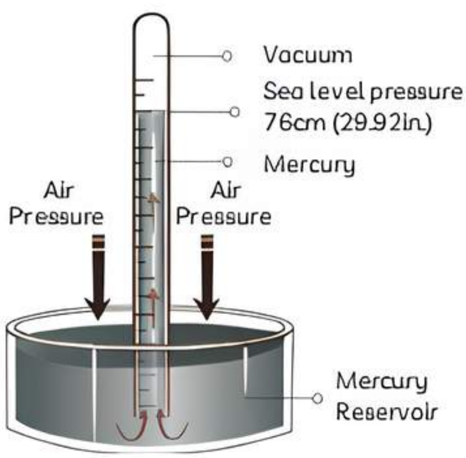
Ideal fluids are not compressible. An ideal fluid (also called Perfect Fluid) is incompressible and has no viscosity. Ideal fluids do not exist but sometimes it is useful to consider what would happen to an ideal fluid in a particular fluid flow problem to simplify the problem.

## CASE BASED Questions (CBQs)

[ 4 & 5 marks ]

**Read the following passages and answer the questions that follow:**

**16.** *Pioneering scientists discovered atmospheric pressure (also known as barometric or air pressure) in the $17^{th}$ century, and determined a startling new fact—that air has weight. Evangelista Torricelli, one of the first to discover atmospheric pressure, once said, "We live submerged at the bottom of an ocean of the element air." The Earth's gravitational field is pulling on air, and this pull, or "pressure" of air, is called atmospheric pressure. Torricelli also went on to develop the mercury barometer, an instrument used to measure atmospheric pressure.*



(A) The speed of blood flow in the human heart is greater in the arteries (broad) than in the capillaries (narrow). However, the equation of continuity ($av$ = constant) appears to predict that the speed should be greater in the capillaries. How are you going to deal with this apparent inconsistency?

(B) Why is blood pressure more in the human body at the feet than at the brain?

(C) What happens when a capillary tube of insufficient length is dipped in a liquid?

**Ans.** (A) In the human heart, the capillaries are many more than arteries. Therefore, the total area of the cross-section of the capillaries is greater than that of the arteries. In accordance with the equation of continuity

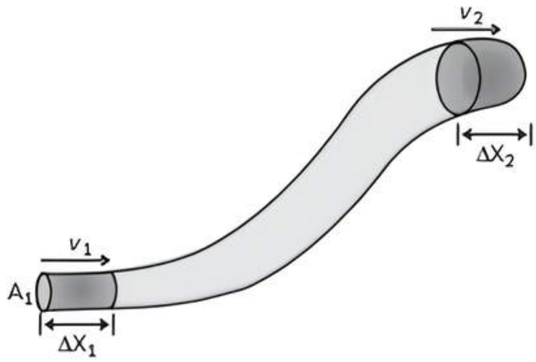
*i.e.*, $av$ = constant,

as the total effective area ($a$) of the capillaries is large, the speed ($v$) of blood flow is small.

(B) The height of the blood column in the human body is more at the feet than at the brain as pressure is directly dependent on the height of the column, so the pressure is more at the feet than at the brain.

(C) When a capillary tube of insufficient length is dipped in a liquid, the radius of curvature increases so that $hr$ = constant. That is pressure on the concave side becomes equal to the pressure exerted by the liquid column so the liquid does not overflow.

**17.** *The transport of some quantities, such as fluid or gas, is described by the continuity equation. The equation describes how a fluid conserves mass while moving. The continuity equations conserve many physical phenomena such as energy, mass, momentum, natural quantities, and electric charge. The continuity equation is useful for determining the flow of fluids and their behaviour as they move through a pipe or hose. The Continuity Equation is used on tubes, pipes, rivers, ducts with flowing fluids or gases, and many other structures. The continuity equation can be expressed in an integral form and applied in a finite region, or it can be expressed in a differential form and applied at a point.*




*Bernoulli's theorem, in fluid dynamics, relation among the pressure, velocity and elevation in a moving fluid (liquid or gas), the compressibility and viscosity of which are negligible and the flow of which is steady, or laminar. Bernoulli's principle is applicable to those non-viscous liquids which have laminar or streamlined flow. It means that a liquid in which its particles exert no force on each other i.e. the speed of all particles of the liquid is the same.*

(A) Bernoulli's principle is based on the conservation of:
- (a) momentum
- (b) energy and momentum both
- (c) mass
- (d) energy

(B) Water is flowing through a horizontal pipe in a streamline flow, at the narrowest part of the pipe:
- (a) both pressure and velocity remain constant.
- (b) velocity is maximum and pressure is minimum.
- (c) both the pressure and velocity are maximum.
- (d) both the pressure and velocity are minimum.

(C) In houses, away from municipal water tanks often find it difficult to get water on the top floor. This happens because:
- (a) water wets the pipe.
- (b) the pipes are not of uniform diameter.
- (c) the viscosity of water is high.
- (d) of loss of pressure during the flow of water.

(D) In which of the following types of flows is Bernoulli's theorem strictly applicable?
- (a) streamline and rotational
- (b) turbulent and rotational
- (c) turbulent and irrotational
- (d) streamline and irrotational

(E) Viscosity of gases:
- (a) decreases with increases in temperature

(b) independent of temperature

(c) increases with an increase in temperature

(d) may increase or decrease depending on nature of gas [Delhi Gov. QB 2022]

**Ans.** (A) *(d) energy*

**Explanation:** The law of energy conservation serves as the foundation for Bernoulli's principle. We compare the total energy of a flowing liquid at various sites flowing under constant pressure difference (including pressure energy, potential energy, and kinetic energy).

**Related Theory:**

➥ *We really gain a pretty clear understanding of the equilibrium between fluid pressure, velocity, and elevation through Bernoulli's equation. There are many uses for this idea. It might be used to approximate both the fluid's pressure and speed. It is also applicable to the theory of acoustics and ocean surface waves. Additional examples include the venturimetre, human propensity to lean towards a moving train, and even how an aeroplane operates.*

(B) *(b) velocity is maximum and pressure is minimum.*

**Explanation:** The product of cross section area and velocity is constant in streamline flow (equation of continuity). Thus, the pipe's smallest section has the highest velocity.

Also, we know that the total amount of potential energy, kinetic energy, and pressure energy remains constant according to Bernoulli's theorem. Potential energy is equal at all sites since the pipe is horizontal. Due to the fact that velocity (kinetic energy) is at its highest at the narrowest region of the pipe, pressure (pressure energy) will be at its lowest there.

(C) *(d) of loss of pressure during the flow of water*

**Explanation:** People frequently have trouble getting water on the top level in homes distant from the municipal water tanks.

This occurs as a result of significant water pressure loss due to friction between the water and the pipeline.

(D) *(d) streamline and irrotational*

**Explanation:** The Bernoulli's principle may be used with non-viscous liquids that flow laminarly or smoothly.

It denotes a liquid whose particles do not interact with one another, *i.e.*, the liquid's particles move at the same speed.

Moreover, the liquid must move in streamlines; that is, the liquid running through one pipeline (imaginary pipeline) must not mix with the liquid flowing through another pipeline. This type of flow is streamlined.

Moreover, streamlined flow is the opposite of turbulent flow. Hence, a turbulent liquid will defy Bernoulli's law.

However this streamlined flow won't happen if the liquid is turned. As a result, this kind of liquid will not be subject to Bernoulli's principle.

(E) *(c) increases with an increase in temperature*

**Explanation:** Gas viscosity is directly correlated with temperature. Hence, the viscosity will increase as the temperature rises.

## VERY SHORT ANSWER Type Questions (VSA)

[ 1 mark ]

**18. A capillary tube is dipped first in cold water and then in hot water. Comment on the capillary rise in the second case.**

**[Delhi Gov. QB 2022]**

**Ans.** We know that $h = \dfrac{2S\cos\theta}{r\rho g}$

The surface tension of hot water is lower than that of cold water. Furthermore, the radius of the capillary tube will grow in hot water owing to thermal expansion. Because of both of these factors, the height of capillary rise will be lower in hot water than in cold water.

**19. Fabrics are usually ironed hot through paper to remove paraffin and other fatty spots. Why is this so?**

**Ans.** The capillary ducts in the paper are much smaller than those in the fabric. When the paraffin or other fatty spots are ironed hot, fat gets soaked into the fine ducts in the paper faster and such spots get removed.

**20. Why are straws used to suck soft drinks?**

**[Diksha]**

**Ans.** When we suck through the straw pressure inside the straw becomes less than the atmospheric pressure. Due to the pressure difference, the soft drink rises in the straw and one can take the soft drink easily.

**21. What will be the effect of increasing temperature on the angle of contact and surface tension? [Delhi Gov. QB 2022]**

**Ans.** Surface tension normally decreases with increasing temperature, but the angle of contact rises.

**22. The sap in trees, which consists mainly of water in summer, rises in a system of capillaries of radius $r = 2.5 \times 10^{-5}$ m. The surface tension of sap is $T = 7.28 \times 10^{-2}$ $Nm^{-1}$ and the angle of contact is 0°. Does the surface tension alone account for the supply of water to the top of all trees?**

**Ans.** Capillarity $r = 2.5 \times 10^{-5}$m

$$S.T. = 7.28 \times 10^{-2}\ Nm^{-1},$$

$$g = 9.8\ m/s^2$$

$$\theta = 0°, \rho = 10^3\ kg/m^3$$

$$h = \frac{2S\cos\theta}{r\rho g}$$

$$= \frac{2 \times 7.28 \times 10^{-2} \cos 0°}{2.5 \times 10^{-5} \times 10^3 \times 9.8}$$

$$= \frac{2 \times 728 \times 10^3}{25 \times 98 \times 10^3}$$

$$= 0.597 \approx 0.6\ m$$

**23. Why is it common practice to wet the end of a thread before attempting to thread it through the eye of a needle when sewing?**

**Ans.** When the end of the thread is made wet (with water), a thin water film is formed all around its fibers. The surface of the water film formed contracts due to surface tension and the thread fibers cling together. The thread then offers a reduced cross-section and it becomes easy to put it through the eye of the needle.

## SHORT ANSWER Type-I Questions (SA-I)

[ 2 marks ]

**24. Suppose an office party is taking place on the top floor of a tall building. Carrying an iced soft drink, you step on the elevator, which begins to accelerate downward. What happens to the ice in the drink? Does it rise farther out of the liquid? Sink deeper into the liquid? Or is it unaffected by the motion?**

**Ans.** The acceleration of the elevator is equivalent to a change in the gravitational field, according to the principle of equivalence. If the elevator accelerates downward, one might be tempted to say that the effect is the same as if gravity decreases the weight of the ice cube decreases, causing it to float higher in the liquid. Recall, however, that the magnitude of the buoyant force is equal to the weight of the liquid displaced by the ice cube. The weight of the liquid also decreases with the effectively decreased gravity. Because both the weight of the ice cube and the buoyant force decrease by the same factor, the level of the ice cube in the liquid is unaffected.

**25. Fig (a) and (b) refer to the steady flow of a non-viscous liquid which of the two figures is incorrect? Why?**

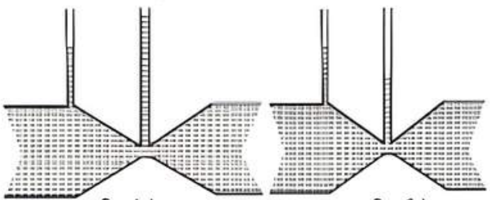


fig (a) fig (b)

**[Delhi Gov. QB 2022]**

**Ans.** The figure in (a) is wrong. According to the equation of continuity, $av$ = Constant, when the area of the cross-section of the tube is less, the velocity of the liquid flow is greater. As a result, the velocity of liquid flow at a tube constriction is greater than at the opposite end of the tube.

According to Bernoulli's Theorem,

$$P + \frac{1}{2}\rho v^2 = \text{Constant.}$$

P is less where $v$ is greater and vice versa.

**26. The surface tension and vapour pressure of water at 20°C is $7.28 \times 10^{-2} Nm^{-1}$ and $2.33 \times 10^3$ Pa, respectively. What is the radius of the smallest spherical water droplet which can form without evaporating at 20°C?**

**[NCERT Exemplar]**

**Ans.** Surface tension of water, S.T. = $7.28 \times 10^{-2}\ Nm^{-1}$

Vapour pressure, P = $2.33 \times 10^3$ Pa

The drop will evaporate if the water pressure is more than the vapour pressure. Let a water droplet of radius $R$ can be formed.

Vapour pressure = Excess pressure in drop

$$P = \frac{2T}{r}$$

$$= 2.33 \times 10^3\ Pa$$

$$r = \frac{2T}{r}$$

$$= \frac{2(7.28 \times 10^{-2})}{2.33 \times 10^3}$$

$$= 6.25 \times 10^{-5}\ m$$

**27. Calculate the minimum pressure required to force the blood from the heart to the top of**

the head at $h$ = 50 cm. Density of the blood is 1.04 g/cm$^{-3}$. Neglect the friction. [Diksha]

**Ans.** Here $h_2 - h_1$ = 50 cm, ρ = 1.04 g/cm$^{-3}$

According to Bernoulli's theorem,

$$P_1 + \rho g h_1 + \frac{1}{2}\rho V_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho V_2^2$$

$$P_2 - P_1 = \rho g(h_2 - h_1)h_1 + \frac{1}{2}\rho(V_2^2 - V_1^2)$$

If, $V_2 = V_1$

$$P_2 - P_1 = \rho g(h_2 - h_1)h_1$$
$$= 1.04 \times 981 \times 50$$
$$= 5.1 \times 10^4 \text{ dyne cm}^{-2}$$

## SHORT ANSWER Type-II Questions (SA-II)

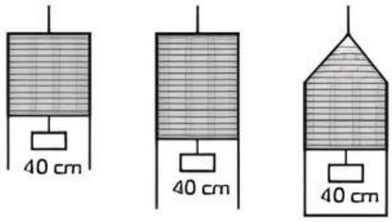
[ **3** marks ]

**28.** **There is a 1 mm thick layer of glycerin between a flat plate of area 100 cm$^2$ and a big plate. If the coefficient of viscosity of glycerin is 1.0kg/ (m-sec), then how much force is required to move the plate with a velocity of 7 cm/sec.** **[NCERT Exemplar]**

**Ans.**
$$F = \eta A \frac{dv}{dx}$$

Here,
$$\eta = 1.0 \text{ kg/m – sec}$$
$$A = 100 \text{ cm}^2 = 10^{-2} \text{ metre}^2$$
$$dv = 7 \times 10^{-2} \text{ m/sec and}$$
$$dx = 1 \text{ mm} = 10^{-3}\text{m}$$
$$F = \frac{1\times10^{-2}\times(7\times10^{-2})}{10^{-3}}$$
$$= 0.7 \text{ N}$$

**29.** **Figure (a) shows a thin liquid film supporting a small weight = 4.5 × 10$^{-2}$ N. What is the weight supported by a film of the same liquid at the same temperature in figure (b) and (c)? Explain your answer physically.**




(a) (b) (c)

**[NCERT Exemplar]**

**Ans.** Length of the film supporting the weight

$$l = 40 \text{ cm} = 0.4 \text{ m}.$$

Total weight supported (*i.e.*, force)

$$W = 4.5 \times 10^{-2}\text{N}.$$
$$T = \text{surface tension} = ?$$

The film has two free surfaces,

So total length,

$$L = 2l = 2 \times 0.4 \text{ m}.$$
$$T = \frac{\text{Force}}{\text{Length}}$$
$$= \frac{4.5\times10^{-2}}{2\times0.4}$$
$$= 5.625 \times 10^{-2} \text{ Nm}^{-1}$$

As the liquid is the same for all cases (a), (b) and (c) and the temperature is also the same. Therefore, surface tension for cases (b) and (c) will also be the same, *i.e.*, 5.625 × 10$^{-2}$ Nm$^{-1}$.

In Figures (b) and (c), the length of the film supporting the weight is also the same as that of (a), hence the total weight supported in each case is 4.5 × 10$^{-2}$ N.

**30.** **Derive an expression for the excess pressure inside a liquid drop.** **[Diksha]**

**Ans.** Excess pressure inside a liquid drop.

Consider a spherical liquid drop of radius R. Let σ be the surface tension of the liquid. Due to its spherical shape, there is an excess pressure P inside the drip over that on the outside. This excess pressure acts normally outward.

Let the radius of the drop increase from R to R + ΔR under the excess pressure P.

Inside surface area = $4\pi r^2$

Final surface are = $4\pi(R + \Delta R)^2$

(Neglecting smaller terms we have)

$$= 4\pi r^2 + 8\pi r\,\Delta R$$

Increase in surface area

(neglecting smaller terms we have)

$$= 4\pi r^2 + 8\pi r\,\Delta R$$

Increase in surface area

$$4\pi r^2 + 8\pi r\,\Delta R - 4\pi r^2 = 8\pi r\,\Delta R$$

Work done in enlarging the drop

= Force × distance

= Pressure × Area × Distance

$$P \times 4\pi r^2\,\Delta R = 8\pi r\,\Delta R\,\sigma$$

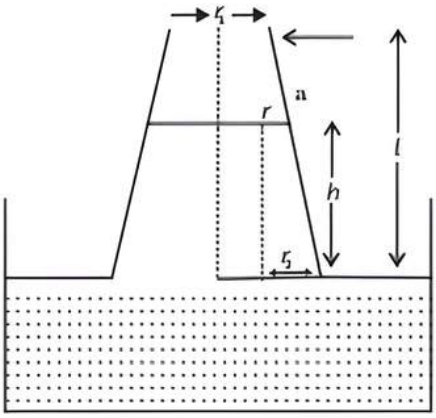
Excess pressure $P = \dfrac{2\sigma}{R}$

## LONG ANSWER Type Questions (LA)

[ 4 & 5 marks ]

**31.** A tube of the conical bore of length 10 cm is just dipped inside the water. The diameters of the upper and lower ends are 0.04 cm and 0.06 cm respectively. Find the height to which the liquid rises in the tube. Given surface tension of water is 70 dyne $cm^{-1}$ and the angle of contact 0°?

**Ans.** According to the question, a conical tube of length $l$ dipped inside water.




Let $r_1$ and $r_2$ be the radii of the upper and lower ends of the tube. Suppose that water rises to a height $h$ in the tube. Let $r$ be the radius of the tube at the meniscus of the water.

Now, $$h = \frac{2T\cos\theta}{r\rho g}$$

Here, T = 70 dyne $cm^{-1}$, θ = 0°, ρ = 1 $gcm^{-3}$,

$$g = 980 \text{ cms}^{-1}$$

$$h = \frac{2 \times 70 \times \cos 0°}{r \times 1 \times 980}$$

$$h = \frac{1}{7r}$$

$$r = \frac{1}{7}h$$

Suppose that the wall of the conical tube makes an angle α with the vertical.

Then, $$\tan\alpha = \frac{r_2 - r_1}{l} = \frac{r - r_1}{l - h}$$

Here, $$r_1 = \frac{0.04}{2} = 0.02 \text{ cm},$$

$$r_2 = \frac{0.06}{2} = 0.03 \text{ cm}.$$

$$l = 10 \text{ cm}$$

$$\frac{0.03 - 0.02}{10} = \frac{r - 0.02}{10 - h}$$

$$(10-h) \times 0.001 = r - 0.02$$

$$0.001h + r - 0.03 = 0$$

Substituting the value of $r$ in the above equation we have,

$$0.001h + \frac{1}{7h} - 0.03 = 0$$

Or $$7h^2 - 210h + 1000 = 0$$

$$h = \frac{210 \pm \sqrt{(210)^2 - 4 \times 7 \times 1{,}000}}{14}$$

$$= \frac{210 \pm 126.9}{14}$$

$$h = 24.06 \text{ or } 5.94 \text{ cm}$$

**32.** Surface tension is exhibited by liquids due to the force of attraction between molecules of the liquid. The surface tension decreases with an increase in temperature and vanishes at boiling point. Given that the latent heat of vaporization for water level ≌ 540 kcal. $Kg^{-1}$, the mechanical heat equivalent of heat J = 4.2 J $cal^{-1}$, the density of water ρ = $10^3$ kg/L Avogadro's No. $N_A$ = 6.0 × $10^{26}$ K $mole^{-1}$ and the molecular weight of water $M_A$ = 18 kg for 1 K mole.

(A) Estimate the energy required for one molecule of water to evaporate.

(B) Show that the intermolecular distance for water is $d = \left[\frac{MA}{NA}, \frac{1}{\rho\omega}\right]^{\frac{1}{3}}$ and find its value.

(C) 1 g of water in vapour state at 1 atm occupies 1601 $cm^3$. Estimate the intermolecular distance at boiling point, in the vapour state.

(D) During vaporisation the molecule overcomes a force F, assumed constant, to go from an intermolecular distance $d$ to $d'$. Estimate the value of F, where $d$ = 3.1 × $10^{-10}$m.

(E) Calculate F/d which is a measure of the surface tension. [NCERT Exemplar]

**Ans.** (A) $L_v = 540 \text{ Kcal Kg}^{-1}$

$= 540 \times 10^3 \text{ cal Kg}^{-1}$
$= 540 \times 4.2 \times 10^3 \text{ J Kg}^{-1}$

Energy required to evaporate 1 kg water

$= L_v$ Kcal.

Energy required to evaporates $M_A$ Kg of Water $= L_A M_A$ Kcal.

In $M_A$ Kg Number of molecules $= N_A$

∴ Energy required to evaporate 1 molecule

$$= \frac{L_V M_A}{N_A}$$

$$U = \frac{L_V M_A}{N_A} \text{Kcal}$$

$$= \frac{L_V M_A \times 10^3 \times 4.2 \text{J}}{N_A}$$

$$= \frac{18 \times 540 \times 10^3 \times 10^3 \times 4.2 \text{J}}{6 \times 10^{26}}$$

$= 12.6 \times 540 \times 10^{-23}$

$= 6.8 \times 10^{-20}$ J

(B) Let the water molecules be point size and separated by distance '$d$' from each other.

Volume of $N_A$ molecules

$$= \frac{\text{Mass of } N_A \text{ molecule}}{\text{density}}$$

$$= \frac{M_A}{\rho\omega}$$

Volume occupied by 1 molecule

$$= \frac{M_A}{\rho\omega N_A}$$

Volume occupied by 1 molecule

$$d^3 = \frac{M_A}{\rho\omega N_A}$$

$$d = \left[\frac{M_A}{\rho\omega N_A}\right]^{\frac{1}{3}}$$

1 g (*i.e.*, $10^{-3}$ Kg) of vapour occupies volume

$= 1601 \text{ cm}^3$
$= 1601 \times 10^{-6} \text{ m}^3$.

(C) 1 Kg of vapour occupies the volume

$= 1601 \text{ cm}^3$
$= 1601 \times 10^{-3} \text{ m}^3$.

18 Kg of vapours occupies the volume

$= 18 \times 1601 \times 10^{-3} \text{ m}^3$.

18 Kg of water $= 6 \times 10^{26}$ molecules

∴ $6 \times 10^{26}$ molecules occupy the volume

$$= \frac{18 \times 1601 \times 10^{-3}}{6 \times 10^{26}}$$

$$d'^3 = \left[\frac{18 \times 1601 \times 10^{-3}}{6 \times 10^{26}}\right]$$

$= [3 \times 1601 \times 10^{-29}]$

$d'^3 = (3 \times 1601) \times 10^{-30}$

$= (36.3 \times 10^{-10})$ m

(D) Work done to change the distance between molecules from $d$ to $d'$ or required work done

$= F(d - d)$

Required work done $= F(d' - d) = 6.8 \times 10^{-20}$

Energy required evaporating 1 molecule

$F(d' - d) = 6.8 \times 10^{-20}$

$F(36.3 \times 10^{-10} - 3.1 \times 10^{-10}) = 6.8 \times 10^{-20}$

$F \times 33.2 \times 10^{-10} = 6.8 \times 10^{-20}$

$$F = \frac{6.8 \times 10^{-20}}{3.32 \times 10^{-10}}$$

$= 0.205 \times 10{-}10$

$F = 2.05 \times 10^{-11}$ N

(E) Surface Tension $= \dfrac{F}{d}$

$$= \frac{2.05 \times 10^{-11}}{3.1 \times 10^{-10}}$$

$= 6.6 \times 10^{-2}$ N/m

$\sigma = 6.6 \times 10^{-2}$ N/m

**33. (A) What do you mean by the coefficient of linear expansion and coefficient of cubical expansion? Derive their relation between them.**

**(B) The coefficient of volume expansion of mercury is $5.4 \times 10^{-4}$°C. What is the fractional change in its density for 80°C rise in temperature? [Diksha]**

**Ans.** (A) Coefficient of linear expansion is the change in length per unit change in temperature and the coefficient of cubical expansion is the change in volume per unit change in temperature.

Let original Length $= l$

Change in length $= \Delta l$

original Volume $= V$

Change in volume $= \Delta V$

Initial Volume $= l^3$

Final Volume $= (V + \Delta V) = (l + \Delta l)^3$

Change in volume = final volume − initial volume

$$\Delta V = l^3 + \Delta l^3 + 3l\,\Delta l(l + \Delta l) - l^3$$

$\Delta l^3$ is neglected because it is too small.

$$\Delta\omega = 3l^2\Delta l + 3l\Delta l^2$$

Also ignoring $3l\Delta l^2$.

Coefficient of volumetric expansion

$$\gamma = \frac{\Delta v}{v} \times \Delta t$$

$$= \frac{3l^2\Delta l}{l^3} \times \Delta t = \frac{3\Delta l}{l} \times \Delta t$$

Let, $\alpha = \frac{3\Delta l}{l} \times \Delta t$

So, $\gamma = 3\alpha$

(B) Here, $\gamma = 5.4 \times 10^{-4}$°C, $\Delta T = 80$°C

Let there be $m$ grams of mercury and its initial volume be V. Suppose that the volume of the mercury becomes V after a rise of temperature of 80°C. Then,

$$V_1 = V(1 + \gamma\Delta T)$$
$$V = (1 + 5.4 \times 10^{-4} \times 80)$$
$$V_1 = 1.0432\ V$$

Initial density of the mercury, $\rho = \frac{m}{V}$

Final density of the mercury,

$$\rho_1 = \frac{m}{V_1} = \frac{m}{1.0432V} = \frac{\rho}{1.0432} = 0.9586\rho$$

Therefore, the fractional change in the value of density of mercury,

$$\frac{\rho - \rho}{\rho} = \frac{\rho - 0.9586\rho}{\rho} = 0.0414\rho$$

## NUMERICAL Type Questions

**34. There is a 1 mm thick layer of glycerin between a flat plate of area 100 cm$^2$ and a big plate. If the coefficient of viscosity of glycerin is 1.0 kg/(m-sec), then how much force is required to move the plate with a velocity of 7 cm/sec. (2m)**

**Ans.** $F = \eta A \frac{dv}{dx}$

Here, $\eta = 1.0$ kg/(m-sec),

$A = 100\ cm^2 = 10^{-2}\ meter^2$,

$dv = 7 \times 10^{-2}$ m/sec

and $dx = 1\ mm = 10^{-3}\ m$

$$F = \frac{1\times10^{-2}\times(7\times10^{-2})}{10^{-3}} = 0.7\ N$$

**35. Terminal velocity of a copper ball of radius 2 mm through a tank of oil at 20°C is 6.0 cm/s. Compare the coefficient of viscosity of the oil. Given $\rho_{cu} = 8.9 \times 10^3$ kg/m$^3$, $\rho_{oil} = 1.5 \times 10^3$ kg/m$^3$.**

**[Delhi Gov. QB 2022] (2m)**

**Ans.** As we know,

Terminal velocity,

$$v_t = \frac{2}{9}\left[\frac{g(\sigma - \rho)r^2}{\eta}\right]$$

$$\eta = \frac{2}{9}\left[\frac{9.8\times(9.8\times10^3 - 1.5\times10^3)(2\times10^{-3})^2}{6\times10^{-2}}\right]$$

$$= 1.08\ kg\ m^{-1}\ s^{-1}$$

**36. Two equal drops of water are falling through the air with a steady velocity of 10 cm/s. If the drops recombine to form a single drop, what would be their terminal velocity? (3m)**

**Ans.** Terminal velocity $v \propto r^2$

Let $v_1$ and $v_2$ be the terminal velocities of drops before and after the combination, respectively.

Hence, $$\frac{V_1}{V_2} = \frac{r^2}{R^2} \quad \text{...(i)}$$

(Where, R = radius of combined drop)

Further, $$2\times\left(\frac{4}{3}\pi R^3\right) = \left(\frac{4}{3}\pi R^3\right)$$

i.e., $$\left(\frac{r}{R}\right)^2 = \frac{1}{(2)^{\frac{2}{3}}}$$

From equations (i) we get,

$$V_2 = V_1\left(\frac{r}{R}\right)^2$$

$$= 10\times(2)^{\frac{2}{3}}$$

$$= 15.9\ cm/sec$$